Sufficient Statistics for Economic Mobility: When Do Measures Agree?

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A “proliferation of measures”

- Researchers measure economic mobility
    - ... in many outcomes (e.g., incomes, wealth, education, health, ...)
    - ... for many uses (e.g., normative analysis, policy evaluation, validating models, . . .)
    - ... for many comparisons (e.g., cross-country, regions, history, . . .)

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- Different measures often produce consistent rankings of mobility (e.g., Katz and Krueger 2017, Berman 2022, Deutscher and Mazumder 2023)

- But not always: long-run US trends (Jácome, Kuziemko & Naidu, 2025); cross-CZ w/in US (Chetty et al., 2014); cross-country (Blanden 2013) ▸ CZs

A Puzzle: 20th Century U.S. Intergenerational Mobility

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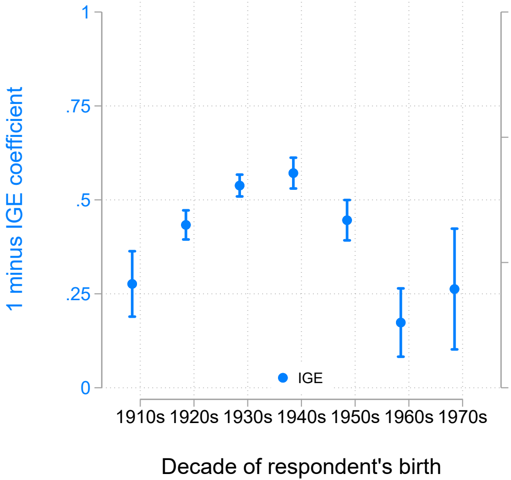

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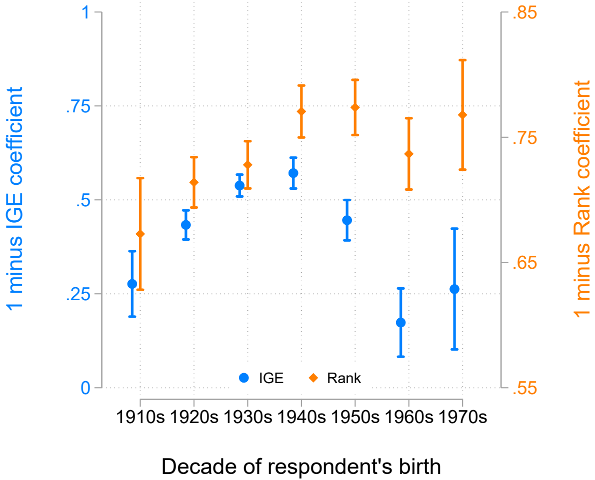

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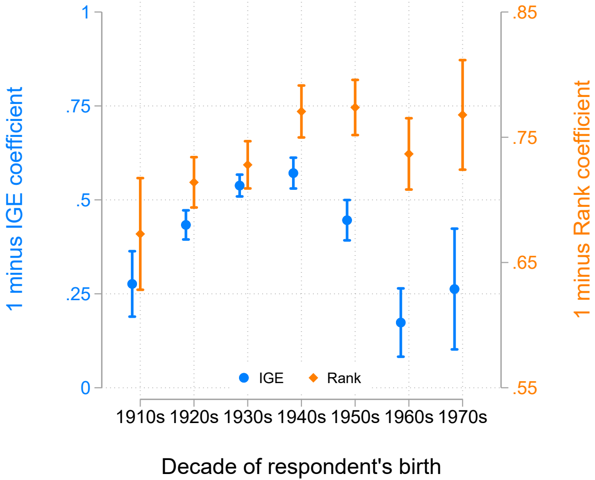
Two fundamental questions:

1. When *should* we expect measures to agree?
2. What does disagreement *tell us*?



This Paper Provides Answers

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    - Concordance order: parent-child ranks are *everywhere* more aligned

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    - Fixing marginal distributions of income $\longrightarrow$ relative mobility

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2. If economies ranked by concordance, *all* standard mobility measures agree
   - Observe ranking in one measure, know all rankings: measures as ordinal substitutes
   - Fixing marginal distributions of income ⟶ relative mobility
3. Changes in any mobility measure = rank-dependence + changing inequality
   - Rank-based measures: only rank-dependence
   - Level-based (incl. IGE): both ⟶ potential for disagreement
   - Disagreement reflects:
     1. different weightings over regions in joint distribution
     2. different sensitivity to changing inequality

Contributions

1. Unifying framework w/ Concordance as sufficient statistic for mobility measurement
    - Adds to measure comparisons (Deutscher, Mazumder; Berman), axiomatic approaches (Shorrocks; Fields, Ok; D'Agostino, Dardanoni; Cowell, Flachaire), dual of 2D-inequality problem (Atkinson, Bourguignon; Dardanoni)
    - Scope of debate over preferred measures
    - Complement inference (Chetverikov, Wilhelm; Kitagawa, Nybom, Stuhler; Haider, Solon) w/ results for proxies and measurement error

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    - Diagnostic tool for applied researchers

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3. Microfounded w/ structural interpretation in Becker-Tomes-Loury models
    - Sorting measures NAM vs PAM in matching (Anderson, Smith; Gola; Boerma, Tsyvinski, Wang, Zhang) and mating (Chiappori, Costa Dias, Meghir, Zhang)

Setup

Separating Dependence from Marginals: Copulas

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Sklar's (1959) Theorem: Any joint distribution decomposes as:

$$F(Y^K, Y^P) = C\big(F_K(Y^K), F_P(Y^P)\big)$$

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- Copula encodes *all* rank-dependence between generations $\perp\!\!\!\perp$ of marginals ▸ Mass points

- Relative mobility measures alternative summaries of copula

Concordance Order: Visual Intuition ▸ Example II

		Child's Quintile				
		1	2	3	4	5
Par. Quintile	1	0.00	0.00	0.00	0.00	1.00
	2	0.00	0.00	0.00	1.00	0.00
	3	0.00	0.00	1.00	0.00	0.00
	4	0.00	1.00	0.00	0.00	0.00
	5	1.00	0.00	0.00	0.00	0.00

$\longrightarrow$

		Child's Quintile				
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Par. Quintile	1	0.20	0.20	0.20	0.20	0.20
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	3	0.20	0.20	0.20	0.20	0.20
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Increasing Concordance

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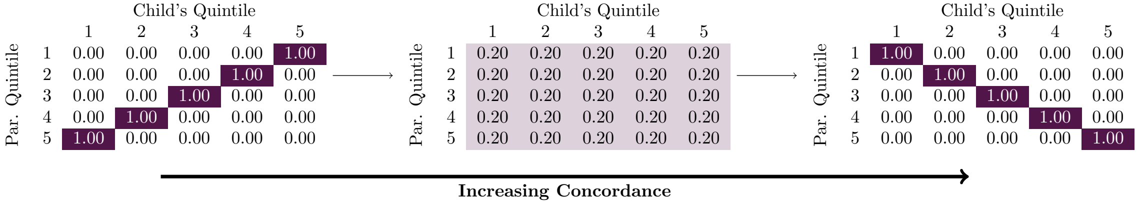

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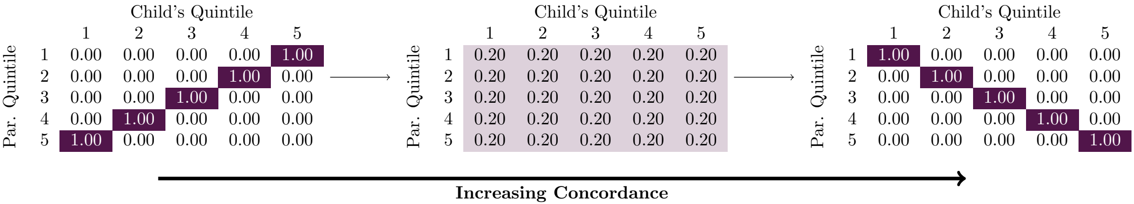

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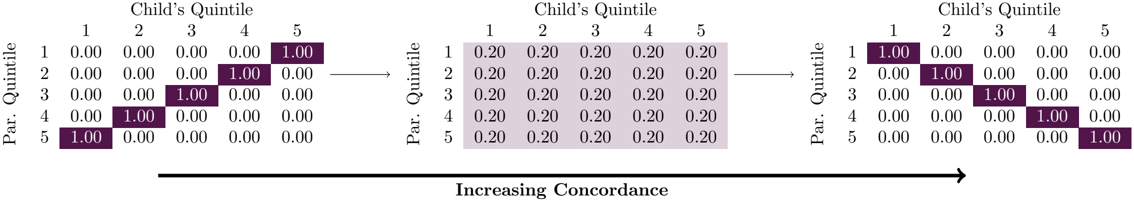

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- “Large” parent values coincide with “large” child values more often
- Pinned down by dependence parameters in parametric families (e.g., ρ in normal)

The Concordance Order

Definition: (Tchen 1980) Economy A is **more concordant** than economy B ($C^A \succeq C^B$) if:

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    "intuitive requirement that children from higher-income families are less likely to have lower incomes" (Chetty et al., 2017); "empirically unlikely transition matrices" (Shorrocks, 1978); "all-or-nothing" (Berman, 2022)

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  - Testable: provide transparent testing procedure
- Higher concordance $\longleftrightarrow$ **lower mobility**

Main Results: Sufficiency

Concordance Orders All Standard Mobility Measures

Props 1-4: Fix marginals. If $C^A \succeq C^B$, then economy A is *less mobile* than B by:

1. Regression/Correlation-based measures:
    - IGE: $\beta^A \geq \beta^B$, Rank-rank slope: $\rho^A \geq \rho^B$, Intergenerational correlation
2. "Local" measures:
    - Conditional expected rank $\longrightarrow$ rotates around $r^* = 0.5$
3. Transition probabilities:
    - diagonal $\uparrow$, off-diagonal $\downarrow$, Directional mobility
4. Axiomatic measures:
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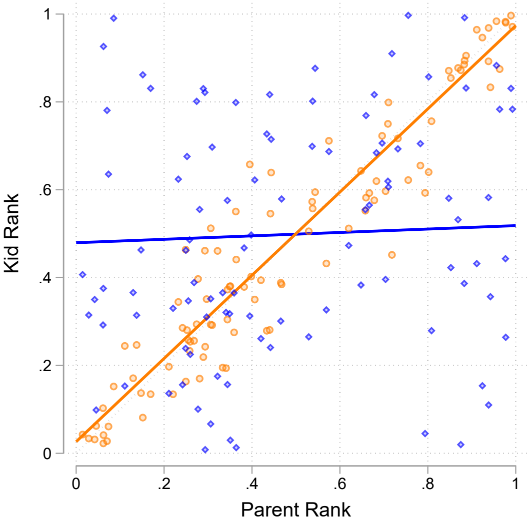
$\longrightarrow$ Measures are ordinal substitutes: observe one ranking, know all others

▸ Regularity Cond.

Why Does Concordance Order These Measures?

Rank-rank slope



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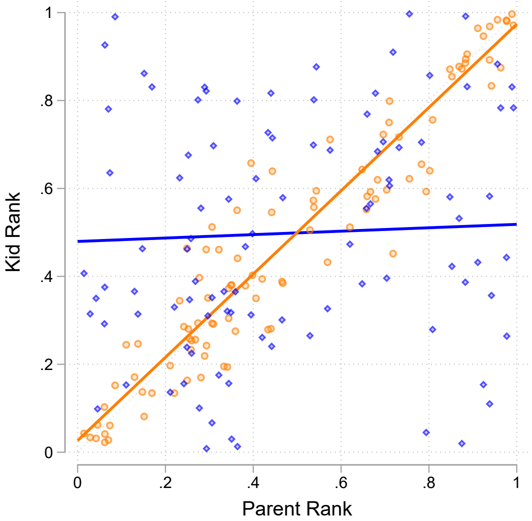

Mobility Measure:

$\rho = 12 \iint C(u,v)\, du\, dv - 3$

$C^A \geq C^B$ everywhere

$\longrightarrow \iint C^A \geq \iint C^B$

$\longrightarrow \rho^A \geq \rho^B$

(Lehmann 1966; Tchen 1980)

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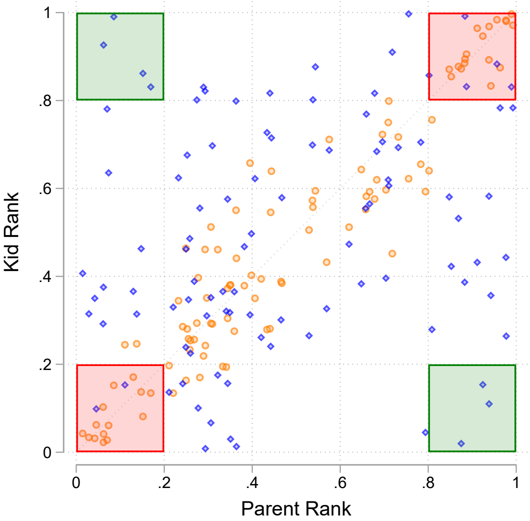

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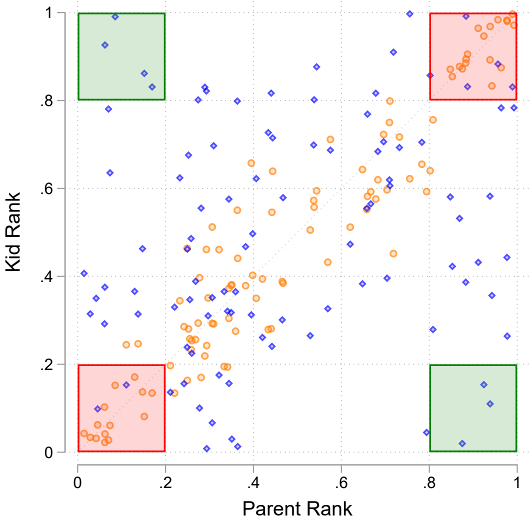

Mobility Measure:

$$\Pr(R^K > \tau \mid R^P > \tau) = \frac{1-2\tau+C(\tau,\tau)}{1-\tau}$$

If $C^A \geq C^B$ everywhere

$$\longrightarrow C^A(\tau,\tau) \geq C^B(\tau,\tau)$$

$$\longrightarrow \text{TP}^A \geq \text{TP}^B$$

(Copula definition, simplest cases!)

Concordance is Pigou-Dalton for Mobility

- Inequality measurement (Atkinson, 1970)
  1. Elementary operation: Pigou-Dalton transfer
  2. Partial order: Lorenz dominance (SOSD)
  3. Sufficiency: SOSD $\longrightarrow$ all transfer-sensitive measures agree

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- Mobility measurement (Today)
  1. Elementary operation: Concordance Exchanges
     - swap Y_i^K and Y_j^K if $Y_i^K < Y_j^K$, but $Y_j^P > Y_i^P$ $\longrightarrow$ increase concordance
  2. Partial order: Concordance
     - Discuss implications & completions
  3. Sufficiency: Concordance $\longrightarrow$ all exchange mobilty measures agree

Analogues in SWF for 2D ineq. (Atkinson-Bourguignon) & Perm. Income ineq. (Dardanoni)

U.S. Mobility Trends
&
Formal Test

Empirical Application: 20th Century U.S. Mobility

Datasets: Replicate Jácome, Kuziemko & Naidu, 2025

- Pool 15 U.S. surveys (ANES, GSS, PSID, OCG, NLS, ...) with respondent income + father's occupation (large sample of intergenerational links) ▶
- Impute parental income from Census using occupation × race × region (using census)
- U.S. Born, age 30-50 (mitigate life-cycle bias), 1910s-1970s birth cohort (YoB)
- 56,102 father-son pairs

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Theory predicts:

- Rank-based measures should **co-move** , but other measures may diverge (rising inequality)

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Construct cardinal measure M_λ for (approx) concordance order

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- Mix perfect immobility (Hoeffding bound M) and perfect mobility ($C^{\mathbb{I}}$)

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- Parallels PAM approach in marriage market (Fernández and Rogerson, 2001; Abbott et al., 2019)

Visualizing Approximate Concordance

Construct cardinal measure M_λ for (approx) concordance order

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Return to formal testing procedure

Concordance Mobility over the 20th Century

Wave-pattern

- Long run Mobility rises over the 20th century
- But plateaus/declines after 1960s

Increasing Mobility? Moment Inequality Test of Concordance Order

▸ Details

- Intuition $\approx$ Kolmogorov-Smirnov test for Stochastic Dominance

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Decade A	Decade B	$pval_{A \succeq B}$	$pval_{B \succeq A}$	Interpretation
1910	1920	.231	.034	$C_{1910} \succeq C_{1920}$
1920	1930	.001	.004	Copulas cross
1930	1940	.052	.001	$C_{1930} \succeq C_{1940}$
1940	1950	.142	.358	Fail to reject
1950	1960	.004	.045	Copulas cross
1960	1970	.190	.017	$C_{1960} \succeq C_{1970}$

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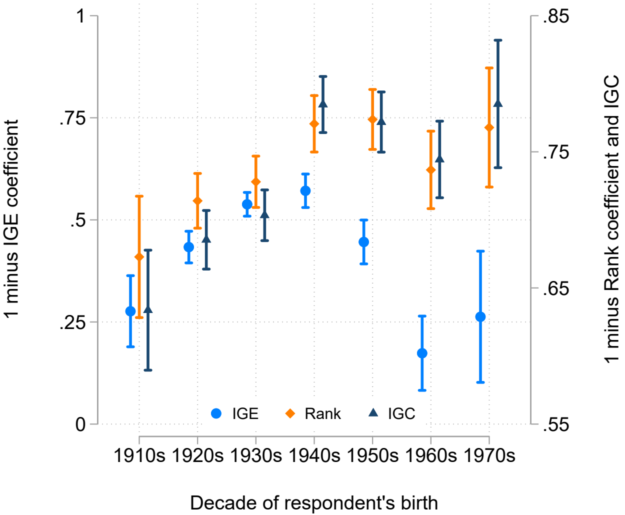
- Some measure disagreement (paper has many measures) noise, others substantive
  $\longrightarrow$ Transition Probabilities disagree when copulas cross

Regression Measures Don’t Comove

▸ Other Measures



- Rank-rank slope inherits wave-like pattern from concordance

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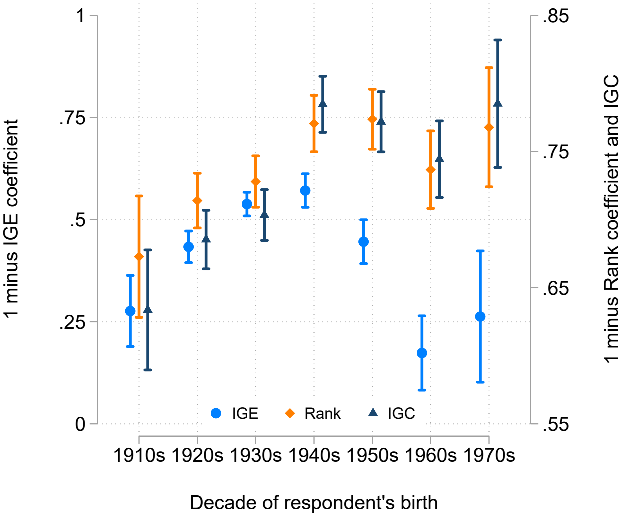

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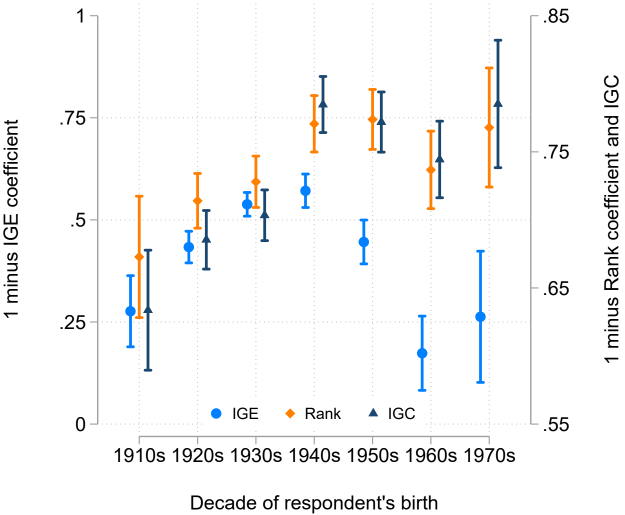

- Rank-rank slope inherits wave-like pattern from concordance
- IGC tracks rank-rank slope (not generic)
- IGE diverges starting 1950s

Why? IGE adds changing inequality to theory results

Extension: When Marginals Differ ▸ e.g.

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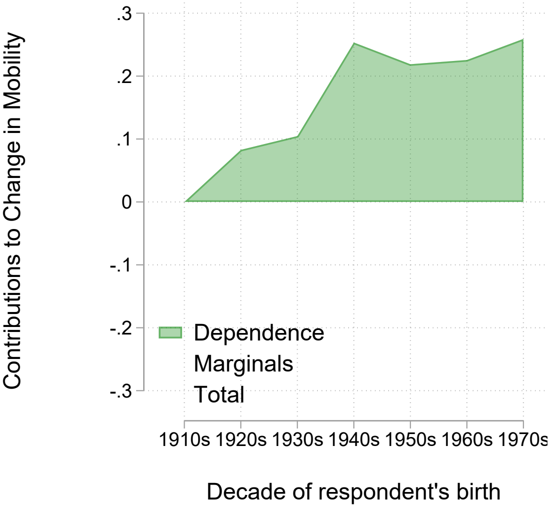
- Order matters $\longrightarrow$ average over fixing copulas at A and B

  (Shapley-Owen-Shorrocks decomposition)

Decomposition: Why Does IGE Diverge?

▸ Other Measures



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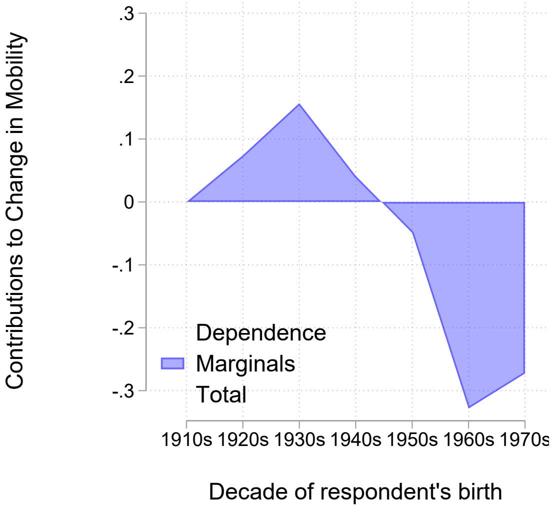

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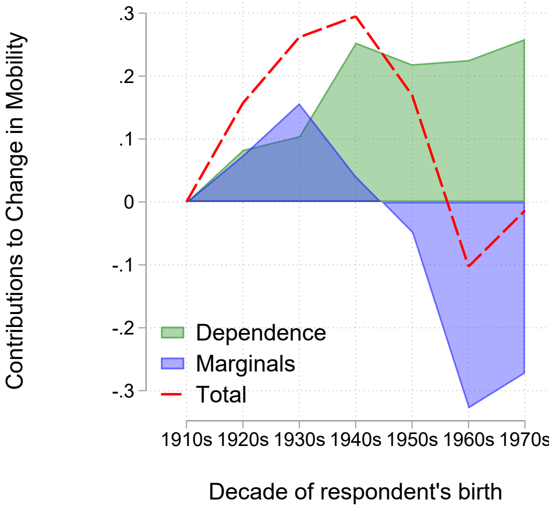

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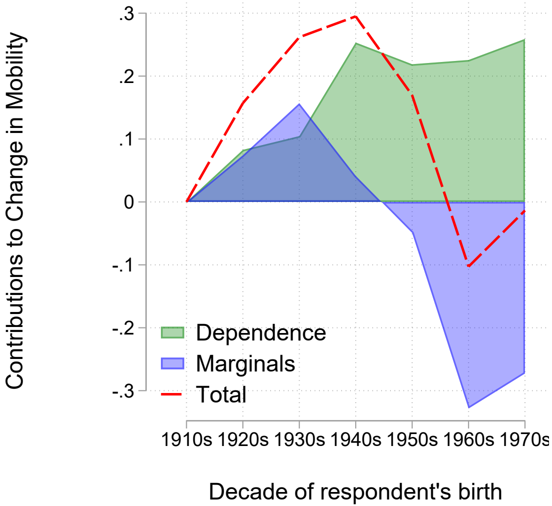

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- Disagreement reflects sensitivity to marginals!

What To Do When Concordance Fails?

Crossing Copulas: 1920s–30s and 1950s–60s

Concordance is a partial order ⟶ not all economies are ranked, e.g.

		Child's Quintile				
		1	2	3	4	5
Par. Quintile	1	0.32	0.24	0.2	0.15	0.08
	2	0.25	0.23	0.21	0.18	0.12
	3	0.19	0.22	0.22	0.21	0.16
	4	0.15	0.18	0.21	0.24	0.22
	5	0.09	0.12	0.16	0.22	0.41

No Concordance Order

⟷

		Child's Quintile				
		1	2	3	4	5
Par. Quintile	1	0.58	0.00	0.00	0.14	0.28
	2	0.00	0.86	0.00	0.00	0.14
	3	0.00	0.00	1.00	0.00	0.00
	4	0.14	0.00	0.00	0.86	0.00
	5	0.28	0.14	0.00	0.00	0.58

- Identical rank-correlation (0.32)

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- If economies not ranked by concordance, can always find two measures:
  (Prop 3. & Corollary 2)
  - A is more mobile than B (1^{st} meas.)
  - B is more mobile than A (2^{nd} meas.)

Source of Disagreement

- Measures expressed as

$$M = \int \int \phi_M \left(Y^K, Y^P \right) f \left(Y^K, Y^P \right) d\,Y^K d\,Y^P,$$

where $\phi_M \left(Y^K, Y^P \right)$: mobility-measure specific intensity

  - TP: Indicator; IGE: $1/\sigma_P^2 (\ln Y^K - \ln \overline{Y}^K)(\ln Y^P - \ln \overline{Y}^P)$

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- For fixed marginals:

$$\Delta M = \int_0^1 \int_0^1 \underbrace{W_M(R^K, R^P; F^K, F^P)}_{\leq 0} \left[C^A(R^K, R^P) - C^B(R^K, R^P)\right] dR^K\, dR^P$$

- Concordance guarantees $C^A - C^B \geq 0 \longrightarrow$ $\Delta M \leq 0 \perp\!\!\!\perp$ of weights ▶

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- Failures $\longrightarrow$ $C^A - C^B$ changes sign on diff. regions, but measures diff. sensitivity
  - Tie-break: Normative (intersecting Lorenz curves) or robustness choice (sensitivity to ϵ-violations)

What To Report When Concordance Fails

Prop 6: Concordance isn't knife-edge!
If $C^A - C^B$ within ϵ of satisfying concordance a measure M agrees up to $L_M \cdot \epsilon$

- L_M small for rank-based measures, larger for IGE and specific TPs
    - Rationalises tighter empirical co-movement of rank-based measures

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Reporting protocol: use test output to localise the violation

- Report local rank–rank slopes on the regions where $C^A - C^B$ changes sign
- Supplement with transition probabilities at standard thresholds

Lets readers with different preferred measures reach their own conclusions

Other Settings
&
Structural Interpretation

No time? Jump to Conclusions

Other Settings ▸ Skip

1. More measures (follow JEL taxonomy, Deutscher & Mazumder) include absolute measures
    - Lorenz mobility (M$^{\underline{c}}$Gee & Ocampo, 2025), Bartholomew jump measure, Goodman, average non-linear income persistence (Arellano, Blundell, Bonhomme, 2017), Katz & Krueger

2. Concordance order maintained under fixed rank comparisons
    - Ranking children by position in parent distributions
    - Subgroup or subnational regions against population distribution

3. Allow for multi-dimensional mobility
    - Whole lifecycle paths (Audoly, M$^{\underline{c}}$Gee, Ocampo & Paz-Pardo, 2025) or many generations

Microfoundation: Becker-Tomes-Loury

- Standard dynastic investment model

$$\max_{C^P, I} U\left(C^P\right) + \delta \mathbb{E}_\theta\left[U\left(C^K\right)\right]$$

subject to

$$C^P = Y^P - I, \qquad C^K = Y^K = W\left(\underbrace{\theta f(I, H^P)}_{H^K}\right)$$

  - Parents choose investment (I) and consumption (C^P); child consumes (C^K)
  - $f(\cdot)$: skill production technology; increasing and (weakly) supermodular
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- When do changes in technology or preferences lead to more mobility?

Investment Expansions $\longrightarrow$ Larger in Concordance ▸ Example

Lemma 1: If investment expands uniformly ▸

$$I^{\star A}\left(Y^{P}\right) \geq I^{\star B}\left(Y^{P}\right) \quad \forall\, Y^{P} \;\longrightarrow\; C^{A} \succeq C^{B}$$

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Mechanisms (Corollary 3) ▸

1. Higher marginal product of investment, $f_I^A(I, H^P) \geq f_I^B(I, H^P)\ \forall\ (H^P, I)$ or subsidy;
2. Higher returns to human capital, $W^A(H^K) \geq W^B(H^K)\ \forall\ H^K$;
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What about heterogeneous policy impacts?

- E.g. school finance equalisation: poor districts ↑, rich districts ↓ (Biasi, 2023)
- $\Delta I^{\star}(Y^P)$ crosses zero ⟶ ΔC changes sign ⟶ crossing copulas
- Rationalises 1920s–30s, 1950s–60s?

Conclusions

Implications For Applied Researchers

A single property—concordance—determines when standard mobility measures agree

For applied researchers:

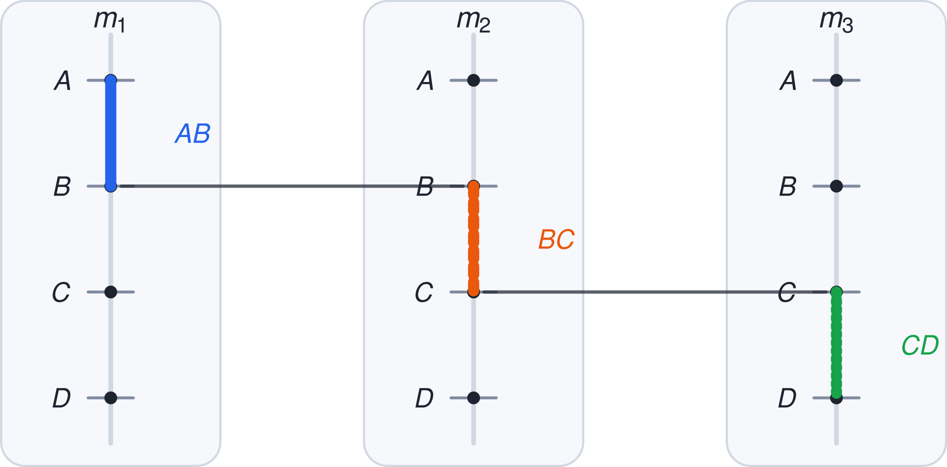
1. When concordance holds: measures are substitutes
    - Pick by data quality, not axiom: transitions for noisy data, IGE for clean panels, rank-rank for cross-country

2. When it fails: disagreement is informative, not noise
    - Test it; report local rank–rank slopes; Lipschitz bounds for ϵ-violations

3. Decomposition as diagnostic: rank-dependence vs. changing inequality

    (reframes IGE-vs-rank-rank divergence)

Chaining Measures



Gives full comparison of A, B, C and D in *any* measure

Details

Mass Points ◂ Back

e.g. from bunching, top-coding, censoring, genuine discreteness $\longrightarrow$ no restrictions on F_K or F_P, but Copula 'indeterminate'

Solutions:

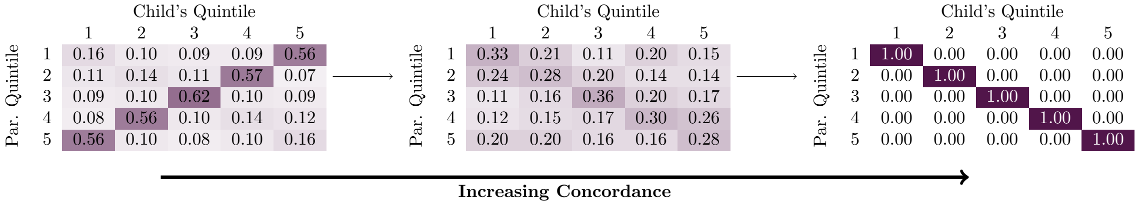
1. Parametric copulas
    - Does violence to discrete data
    - Restrictive

2. Carley (2002) bounds on copulas
    - A lot more math, but flexible
    - Worst-case bounds may not be tight

3. Pick an extension: checkerboard or Piecewise bilinear
    - Propositions 11, 13 in Genest and Nešlehová (2007) show the unique extension preserving concordance order of the original joint-distributions

Concordance: Visual Intuition

◂ Back



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Par. Quintile	1	2	3	4	5
1	0.16	0.10	0.09	0.09	0.56
2	0.11	0.14	0.11	0.57	0.07
3	0.09	0.10	0.62	0.10	0.09
4	0.08	0.56	0.10	0.14	0.12
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$\longrightarrow$

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3	0.11	0.16	0.36	0.20	0.17
4	0.12	0.15	0.17	0.30	0.26
5	0.20	0.20	0.16	0.16	0.28

$\longrightarrow$

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Par. Quintile	1	2	3	4	5
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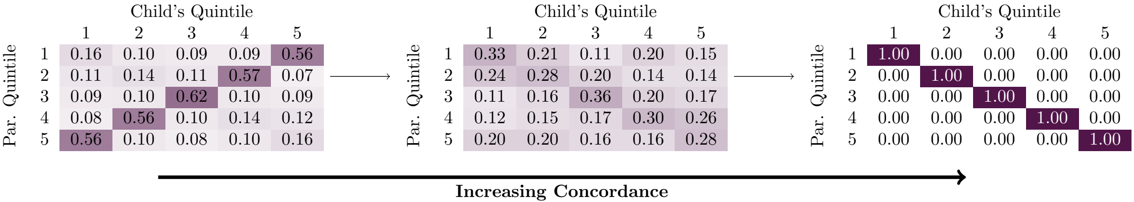
Increasing Concordance

Concordance: Visual Intuition

◂ Back



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Par. Quintile	1	1.00	0.00	0.00	0.00	0.00
	2	0.00	1.00	0.00	0.00	0.00
	3	0.00	0.00	1.00	0.00	0.00
	4	0.00	0.00	0.00	1.00	0.00
	5	0.00	0.00	0.00	0.00	1.00

Increasing Concordance

- Not just extreme cases

Regularity Conditions ◂ Back

Some results require extra structure

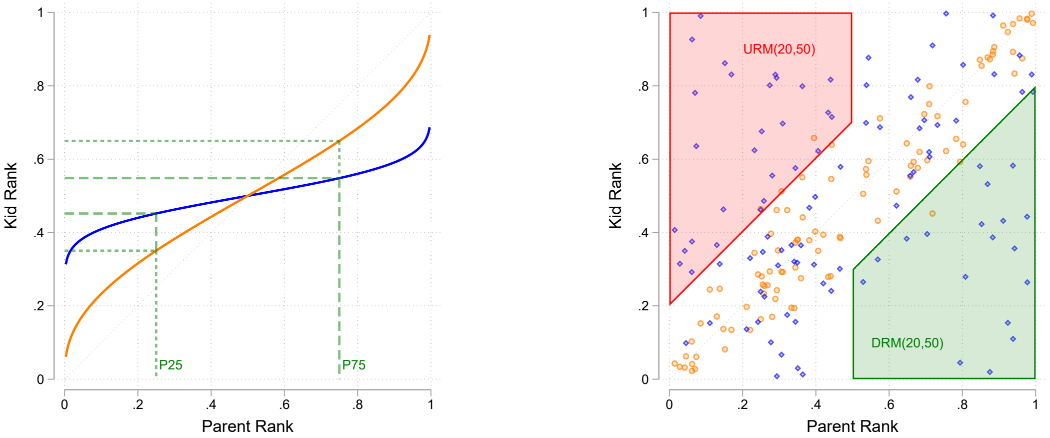
1. Single crossing of conditional densities and continuity (sufficient: TP2 or MLRP)
    - For Non-parametric CER rotation
2. Smooth Anti-Diagonal to Diagonal Exchanges:

$$\Delta C(u_1, v_1) + \Delta C(u_2, v_2) \geq \Delta C(u_2, v_1) + \Delta C(u_1, v_2).$$

- Implies Concordance Order
- Directional Mobility & Shorrocks

Why Does Concordance Order These Measures? Illustration III

◂ Back



Extension: Proxies & Measurement Error ◂ Back

Prop 5: Observe proxy $E^g = H_g(R^g, \varepsilon^g)$ for generation g, H_g increasing, and ε independent of ranks. Let $\widetilde{C}$ denote copula for (E^K, E^P). Concordance ordering preserved:

$$C^A \succeq C^B \quad \longrightarrow \quad \widetilde{C}^A \succeq \widetilde{C}^B$$

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- Covers common proxy cases ▸
    1. Education/occupation as income proxies: $Y \longrightarrow E$
    2. Income and Education as skill proxies: $S \longrightarrow E, Y$

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- Covers common proxy cases ▸
  1. Education/occupation as income proxies: $Y \longrightarrow E$
  2. Income and Education as skill proxies: $S \longrightarrow E, Y$
- Covers flexible measurement error cases $\widetilde{Y} = E = H_g(R^g, \varepsilon^g)$ (Corollary 1) ▸
  1. (non-)Classical measurement error in (log-)incomes (Bound, Brown, Duncan & Rogers 1994)
  2. Measurement error in ranks (Chetverikov & Wilhelm 2023)
  3. Life cycle biases (Kitagawa, Nybom, & Stuhler 2018)

Proxies & Meas Error: Functional Forms ◂ Back

Proxies:

1. Human capital determines incomes $Y = H(E) + \varepsilon$ with $E = H^{-1}(Y - \varepsilon)$
2. Ability determines education and income $E = H_E(S, \varepsilon)$ and $Y = H_Y(S, \varepsilon)$.

Proxies & Meas Error: Functional Forms ◂ Back

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Measurement error:

1. Classical measurement error in incomes
   $$\widetilde{Y}^K = F_K^{-1}(R^K) + \varepsilon^K, \qquad \widetilde{Y}^P = F_P^{-1}(R^P) + \varepsilon^P$$
2. Measurement error in ranks
   $$\widetilde{R}^K = F_{\widetilde{K}}\left(F_K^{-1}(R^K) + \varepsilon^K\right), \qquad \widetilde{R}^P = F_{\widetilde{P}}\left(F_P^{-1}(R^P) + \varepsilon^P\right)$$
3. Life Cycle Biases
   $$\widetilde{Y}^K = G_K\left(F_K^{-1}(R^K) + \mu_{\varepsilon^K} + \sigma_{\varepsilon^K}\varepsilon^K\right), \qquad \widetilde{Y}^P = G_P\left(F_P^{-1}(R^P) + \mu_{\varepsilon^P} + \sigma_{\varepsilon^P}\varepsilon^P\right)$$

Measure specific weights ◂ Back

- Measures expressed as

$$M_M = \int \int \phi_M \left(Y^K, Y^P \right) f \left(Y^K, Y^P \right) d\,Y^K d\,Y^P,$$

  where $\phi_M \left(Y^K, Y^P \right)$: mobility-measure specific intensity

- For fixed marginals, any twice-differentiable measure: $M_M^A - M_M^B$

$$= \int_0^1 \int_0^1 W_M(R^K, R^P; F^K, F^P) \left[C^A(R^K, R^P) - C^B(R^K, R^P) \right] dR^K\, dR^P$$

  where

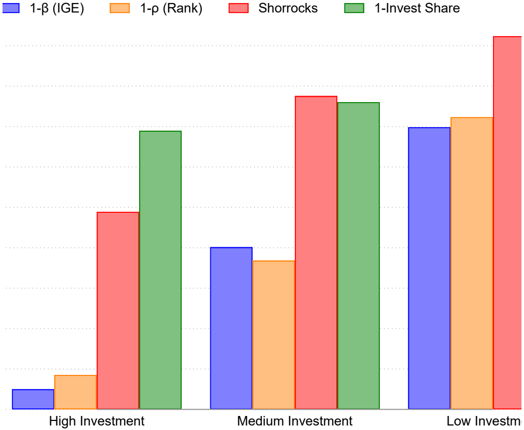
$$W_M(R^K, R^P) = \frac{\phi_{M,12}\left(F_K^{-1}(R^K), F_P^{-1}(R^P)\right)}{f_K\left(F_K^{-1}(R^K)\right)\, f_P\left(F_P^{-1}(R^P)\right)}$$

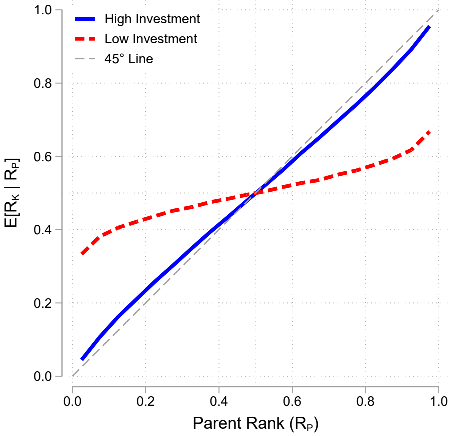
- Works for non-continuous cases also

Investment Expansions Example

◂ Back

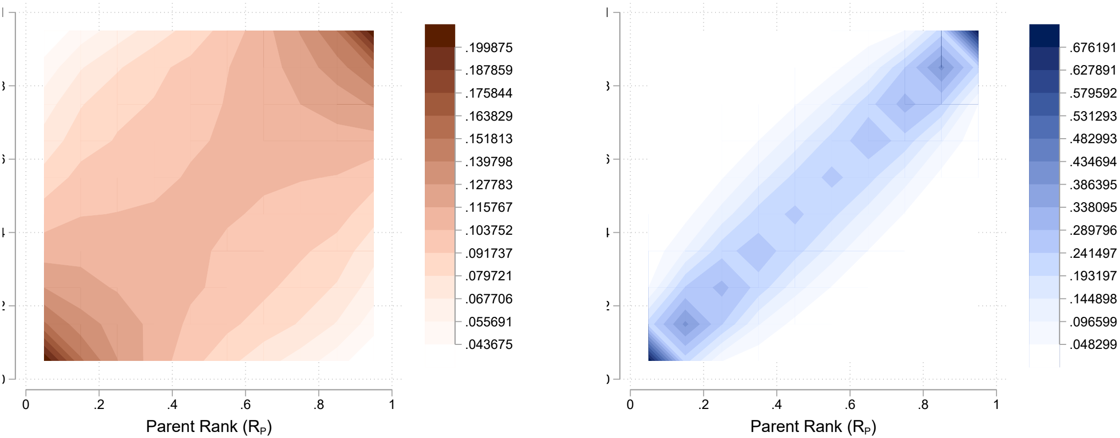
Log-utility and Cobb-Douglas Production





Investment Expansions Example

Back



Proof Sketch: Lemma (Investment $\longrightarrow$ Concordance) ◂ Back

- $I^{*A}(Y^P) \geq I^{*B}(Y^P)$ for all Y^P

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- Under common CDF, rank variables satisfy:

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- But CDF not common, 3 cases:
  1. *Rank-dependent wages:* Integrate over $Y^P \Rightarrow (R^K, Y^P)^A \succeq (R^K, Y^P)^B$
  2. *Common wage function:* FOSD of child incomes + decomposition theorem $\longrightarrow$ concordance $\uparrow$
  3. *Increased returns:* Analogous reasoning applies

Proof Sketch: Corollary (Policy $\longrightarrow$ Investment) ◂ Back

Define value function for investment *I* and 'policy' λ:

$$V(I;\lambda) = U(C^P) + \delta_\lambda\,\mathbb{E}_\theta\big[U(W_\lambda(H^K))\big]$$

subsume subsidy or transfer into *f*

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Key condition (Topkis): Increasing differences holds if

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Verification:

- Production function f is supermodular
- Wage schedule W_λ is strictly increasing
- Standard regularity on GE effects (e.g., multiplicative shifts)

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- Production function *f* is supermodular
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Conclusion: $\lambda' < \lambda \implies I^*(Y^P;\lambda') \leq I^*(Y^P;\lambda)$

Jácome, Kuziemko & Naidu (2025) Replication ◂ Back

15 Pooled Surveys:

- General Social Survey (GSS), American National Election Survey (ANES)
- Panel Study of Income Dynamics (PSID), Occupational Changes in a Generation (OCG)
- National Longitudinal Surveys (NLS): Mature Women, Older Men, Younger Women
- National Survey of Black Americans, National Fertility Survey, others

Sample Construction:

- Inclusion: US-born, ages 30–50, non-missing family income + father's occupation
- Race: White and Black only (other races $<$ 1% pre-1950)
- Weights: Reweighted to census race × sex shares per birth decade

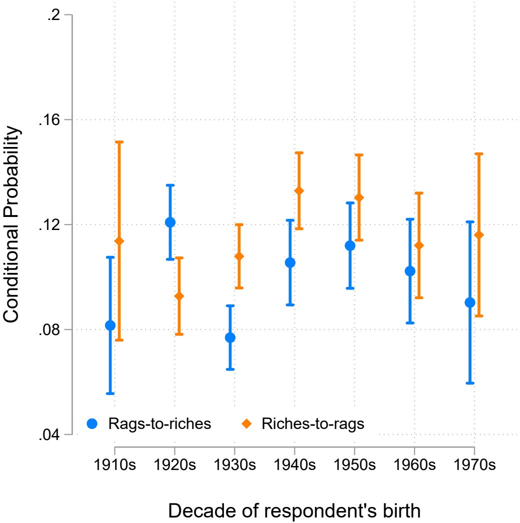
Parental Income Imputation:

- Predict $\widehat{Y}^P$ using 28 harmonized occupation categories × race × South
- Sources: 1901 Cost of Living Survey, 1936 Expenditure Survey, 1940–90 Censuses
- Matched to respondent's $\approx$10th birthday; special adjustments for farmers

Sample Sizes by Birth Decade:

1910s	1920s	1930s	1940s	1950s	1960s	1970s
4,641	11,857	11,426	10,288	9,531	5,691	2,668

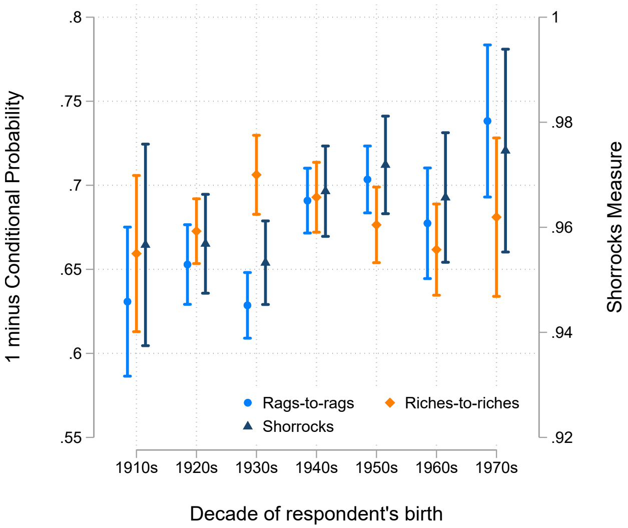
Transition Matrix Measures Co-Move ◂ Back



- Quintile Transition Matrix
- Q1→Q5 & Q5→Q1

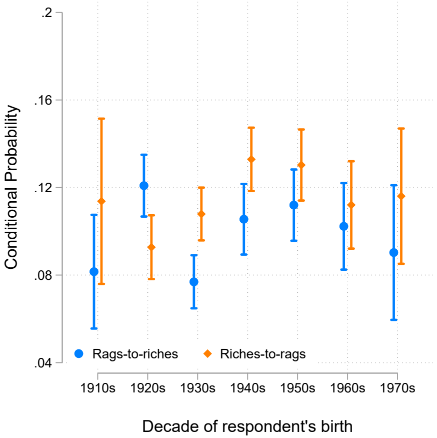
Transition Matrix Measures Co-Move

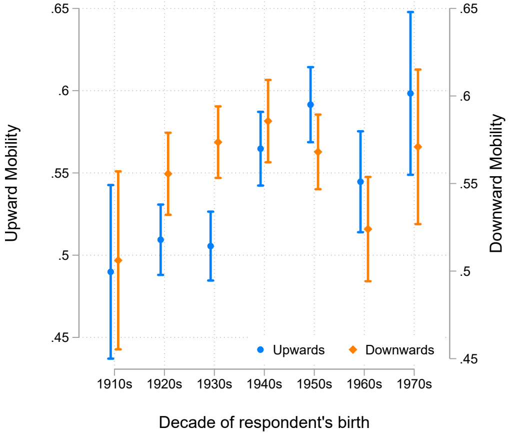
◂ Back



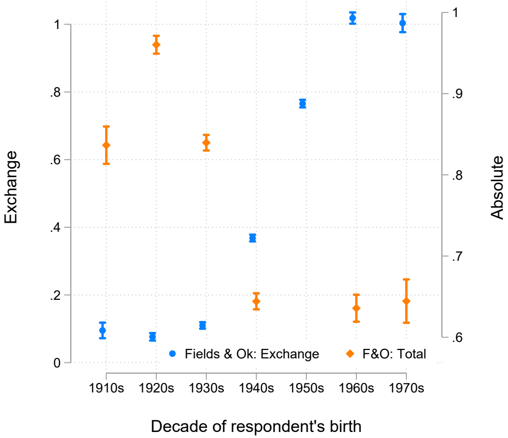
- Quintile Transition Matrix
- Q5→Q5 & Q1→Q1 & Trace

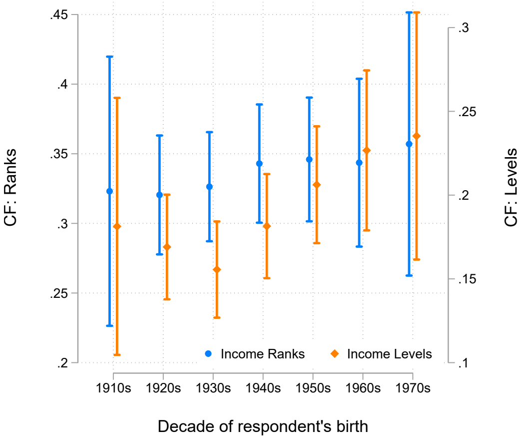
◂ Back


Conditional Probability
.2
.16
.12
.08
.04
Rags-to-riches
Riches-to-rags
1910s
1920s
1930s
1940s
1950s
1960s
1970s
Decade of respondent's birth

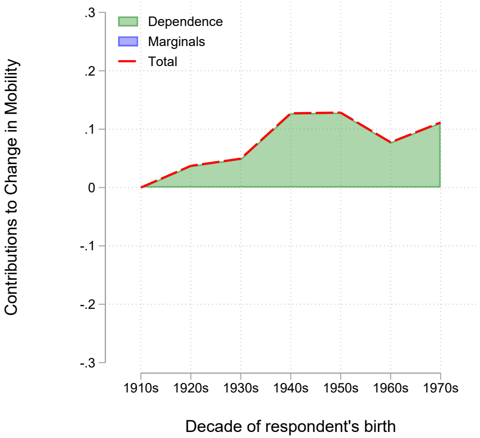

Upward Mobility
.65
.6
.55
.5
.45
Downward Mobility
.65
.6
.55
.5
.45
Upwards
Downwards
1910s
1920s
1930s
1940s
1950s
1960s
1970s
Decade of respondent's birth

Back

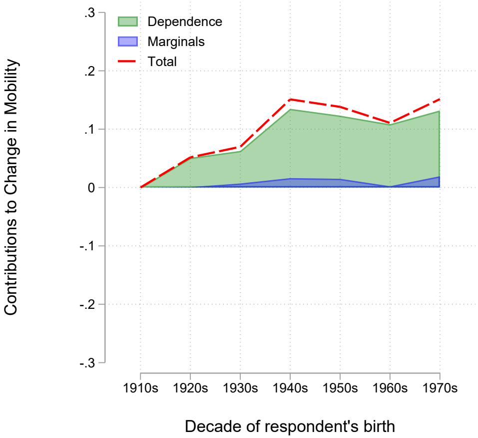

Exchange
Absolute
1
.8
.6
.4
.2
0
1
.9
.8
.7
.6
● Fields & Ok: Exchange
◆ F&O: Total
1910s
1920s
1930s
1940s
1950s
1960s
1970s
Decade of respondent's birth


CF: Ranks
CF: Levels
.45
.4
.35
.3
.25
.2
.3
.25
.2
.15
.1
● Income Ranks
◆ Income Levels
1910s
1920s
1930s
1940s
1950s
1960s
1970s
Decade of respondent's birth

◂ Back



.3
.2
.1
0
-.1
-.2
-.3
Contributions to Change in Mobility
🟩 Dependence
🟦 Marginals
Total
1910s
1920s
1930s
1940s
1950s
1960s
1970s
Decade of respondent's birth



.3
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0
-.1
-.2
-.3
Contributions to Change in Mobility
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Total
1910s
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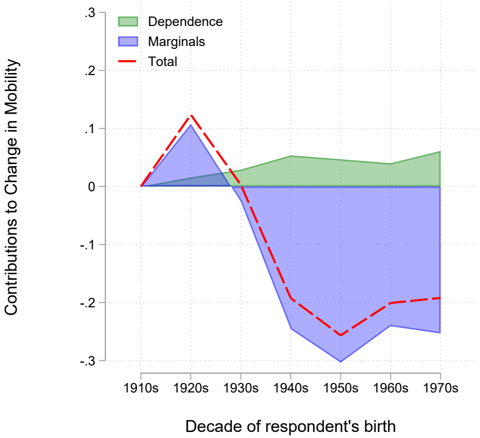
◂ Back


Dependence
Marginals
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Contributions to Change in Mobility
.3
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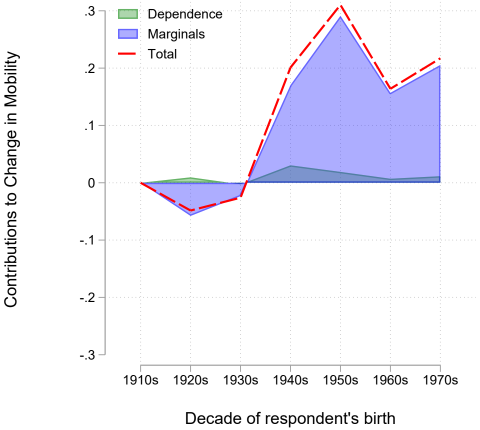
Dependence
Marginals
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.3
.2
.1
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Testing The Concordance Order ◂ Back

Set up as a moment inequality testing procedure (Andrews and Soares, 2010)

- Moments on a grid $\mathcal{G}$:

$$m_g \;=\; C_A(u_g, v_g) - C_B(u_g, v_g), \qquad g = 1, \ldots, G.$$

- Null hypothesis of dominance

$$H_0: \; m_g \geq 0 \;\text{ for all } g = 1, \ldots, G \qquad \text{vs.} \qquad H_1: \; m_g < 0 \;\text{ for some } g.$$

- Test statistic is the minimum of the studentized moments:

$$T_n \;=\; \min_{g=1,\ldots,G} \frac{\widehat{m}_g}{\widehat{s}_g},$$

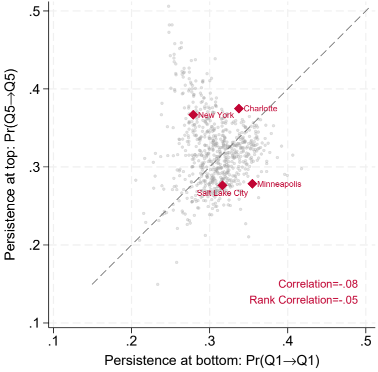
  w/ bootstrap standard error $\widehat{s}_g$

- Test $A \succeq B$ and $B \succeq A$

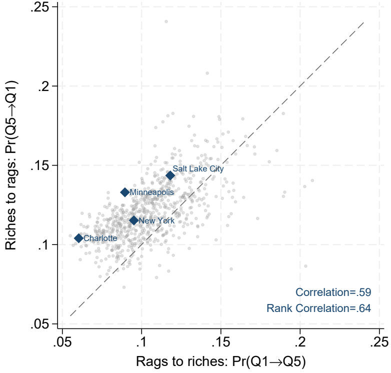
Spatial Mobility: Measure Disagreement Across CZs

◂ Back

Chetty et al. (2014) CZ transition matrices, remapped to within-CZ copulas



Q1→Q1 vs. Q5→Q5 persistence



Q1→Q5 vs. Q5→Q1 tail mobility

- Salt Lake $\succ$ Charlotte on *every* measure...
  - ...but Salt Lake vs. NYC, NYC vs. Minneapolis: depends on measure

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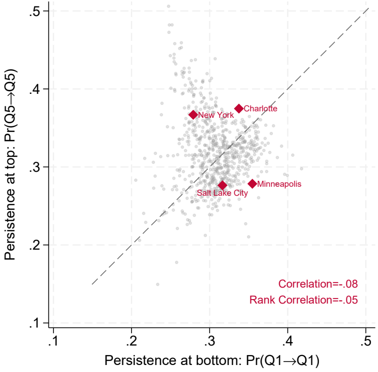

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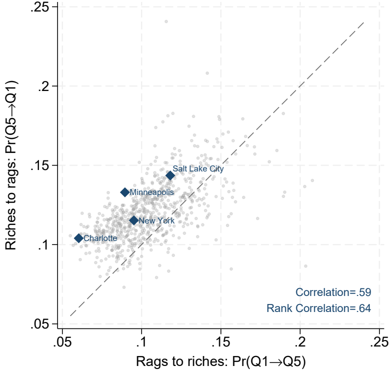

Q1→Q5 vs. Q5→Q1 tail mobility

- Salt Lake $\succ$ Charlotte on *every* measure...
  - ...but Salt Lake vs. NYC, NYC vs. Minneapolis: depends on measure
- Persistence top-quartile$\neq$ in **91%** of CZs; **61%** for the tail-mobility

A Simple Illustrative Example ◂ Back

Separating Dependence & Inequality

Assume Incomes jointly Log-Normal (closed form)

$$\begin{pmatrix} \ln Y^K \\ \ln Y^P \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_K \\ \mu_P \end{pmatrix}, \begin{pmatrix} \sigma_K^2 & \rho\sigma_K\sigma_P \\ \rho\sigma_K\sigma_P & \sigma_P^2 \end{pmatrix}\right]$$

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What do measures depend on?

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- Rank-rank slope: $\mathrm{Corr}(R^K, R^P) = {}^{6}/_{\pi} \arcsin(\rho/2) \approx \rho$ (Kruskal, 1958) *only* dependence!

A Simple Illustrative Example ◂ Back

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- IGE: $\beta = \rho \cdot \frac{\sigma_K}{\sigma_P}$ dependence *and* inequality
    - Changes: $\Delta\beta = \underbrace{(\rho^B - \rho^A) \cdot \frac{\sigma_K^B}{\sigma_P^B}}_{\text{change in rank correlation}} + \underbrace{\rho^A \cdot \left(\frac{\sigma_K^B}{\sigma_P^B} - \frac{\sigma_K^A}{\sigma_P^A}\right)}_{\text{change in inequality}}$

A Simple Illustrative Example ◂ Back

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  - Rising inequality ($\sigma_K/\sigma_P \uparrow$) can dominate IGE even as rank dependence weakens ($\rho \downarrow$)