Sufficient Statistics for Economic Mobility*

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Abstract

A large empirical literature measures intergenerational mobility with intergenerational elasticities, rank-rank slopes, transition matrices, Shorrocks (1978) indices and more. I show that all of these estimands move in the same direction when the underlying parent–child joint distribution becomes more concordant—a dependence concept well known in statistics but unexplored in mobility research. Consequently, seemingly disparate statistics are substitutable: each provides the same summary of more, or less, mobility. I *(i)* prove monotonicity results covering both regression-based and axiomatic indices, *(ii)* extend them to settings with proxy outcomes or measurement error, and *(iii)* embed the concordance order in a Becker-Tomes–Loury human-capital model, illustrating economic forces that increase or decrease mobility uniformly across measures.

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1. Introduction

Empirical research uses a wide range of measures of economic mobility such as rank-rank correlations, intergenerational earnings elasticities, or the Shorrocks-index.[1] In many cases, the results from these measures imply a consistent view of different features of economic mobility. Recent work highlights that many of these measures produce similar evaluations of economic mobility (e.g., Katz and Krueger 2017, Feigenbaum 2018, Berman 2022, Deutscher and Mazumder 2023, and Audoly et al. 2024) despite no unifying framework establishing when agreement is guaranteed. In contrast, Jácome, Kuziemko, and Naidu (2025) document disagreement in the time trends of the intergenerational earnings elasticity and the rank-rank correlation in the U.S. over the 20th century. This paper clarifies when these behaviours arise.

I prove that the concordance order, a single dependence property widely used for ordering joint distributions, links many mobility measures in the economics literature. When the joint parent-child distribution is more dependent in the sense measured by the concordance order, changes to mobility measures share the same sign. This strong notion of dependence explains why empirical estimates of different mobility measures may move together under shifts in the underlying dependence structure. It yields a simple and novel punchline: increasing concordance goes hand in hand with decreasing mobility by any standard measure.

Researchers have proposed various arguments for favouring one common measure over another, including axiomatic justifications, or report a proliferation of different

[1] Researchers use mobility metrics to study the dependence of income across and within generations (e.g., Solon 1992; Chetty et al. 2014b), as well as wealth (e.g., Boserup, Kopczuk, and Kreiner 2018; Fagereng et al. 2020; Fagereng, Mogstad, and Rønning 2021; Audoly et al. 2024), consumption (e.g., Jappelli and Pistaferri 2006), health (e.g., Halliday 2023; Black et al. 2024), education (e.g., Black and Devereux 2011), socio-emotional skills (Attanasio, De Paula, and Toppeta 2025), or occupational prestige (Mazumder and Acosta 2015; Olivetti and Paserman 2015; Song et al. 2020; Haeck and Laliberté 2025). They are used in positive analysis, normative analysis (e.g., Shorrocks 1978; Atkinson and Bourguignon 1982), evaluating the impacts of social policy (e.g., Chetty and Hendren 2018a,b), as well as validating or as targets for structural models of economic behaviour (e.g., Abbott et al. 2019; Bolt et al. 2025; Daruich 2018).

measures to show robustness across measures. The intergenerational elasticity is widely used because it is straightforward to interpret, readily comparable to an established evidence base, and consistent with canonical models of human capital formation (Black and Devereux 2011; Corak 2013), though it is also vulnerable to life-cycle bias and measurement error (Haider and Solon 2006; Nybom and Stuhler 2017). Rank-rank measures are preferred because they are "much more robust across specifications" (Chetty et al. 2014a) and unit-free, aiding comparisons across different locals. Transition matrices, in turn, are valued for their transparency to practitioners (Jäntti and Jenkins 2015) and their capacity to reveal heterogeneity across the joint distribution of parental and child earnings (Black and Devereux 2011), but they can be sensitive to discretization and obscure within-bin dynamics (Cowell and Flachaire 2018). When economies can be ranked in the concordance order, these arguments are redundant: all of these mobility measures deliver the same ranking from least to most mobile.

This paper's primary contribution is to provide a theoretical basis for using the concordance order as a sufficient statistic for ranking economic mobility. Connections between increases in concordance and correlations or supermodular functions are established in statistics (e.g., Lehmann 1966; Tchen 1980). However, these results have not been used to study economic mobility or the relationship between mobility measures. Although this framework directly applies to intergenerational income mobility, it applies equally to intragenerational mobility, earnings dynamics, and other settings. For concreteness, the notation will follow parent–child notation standard in the intergenerational mobility literature.

Importantly, measuring mobility with increased concordance corresponds to economically meaningful changes in behaviour. For example, declining economic mobility may reflect changes in the technology that produces future human capital or shifts in the preferences of parents. Concordance can be microfounded as it naturally characterizes these comparative statics in a tractable model of endogenous human capital investment in the spirit of Becker and Tomes (1979) or Loury (1981).

I provide three sets of results establishing conditions under which commonly used measures of exchange mobility are sufficient statistics for the entire joint-distribution of parent-child outcomes. First, I show that widely used regression or correlation based methods (see Jäntti and Jenkins 2015, for a discussion of their popularity) all decrease in the concordance ordering. Second, I show that features of the transition matrix (such as directional mobility) as well as axiomatic measures of intergenerational mobility (including the seminal contribution of Shorrocks 1978) also decrease in the concordance order. Finally, I show that implications of the concordance order are robust to proxies and measurement error. In doing so, these results strengthen the use of *many* measures of mobility as sufficient statics because it guarantees their agreement. Building on the recent empirical contribution of Jácome, Kuziemko, and Naidu (2025), I document trends in U.S. intergenerational mobility over the 20th century. I show agreement between a wide range of commonly used measures of economic mobility and explain why a small number of measures disagree.

These results extends to other bivariate settings beyond parent-child pairs, but, importantly, also to multivariate outcomes such as multiple generations. I focus on the bivariate joint-distribution: grounding the analysis in the study of parent-child economic mobility. This not only offers a tool to help applied researchers select between measures, but also helps extrapolate between economic mobility measured in different ways.

These sufficiency results show concordance is a powerful tool to rank economic mobility. It is intuitively appealing, directly testable (Denuit and Scaillet 2004; Cebrián, Denuit, and Scaillet 2004) and fully characterizes dependence in many parametric distributions. These include bivariate log-normal (e.g., Solon 1992; Berman 2022); Singh and Maddala (1976) distributions; and archimedian copulas (Callaway, Li, and Murtazashvili 2021) used for modelling income distributions in one or more generations.

Despite this appeal, however, it is not always possible to compare joint distributions in their concordance. For example, shifting mass towards perfect rank-reversals can lead to distributions that cannot be compared in their concordance. In these cases, I show that different mobility measures explicitly weight different parts of the joint distribution with different intensities. Additionally, the concordance order does not hold in levels of income when marginal distributions change over time and place. In these cases mobility measures in levels conflate mobility, economic growth and increases in inequality. Nevertheless, concordance continues to order mobility measures that operate on the copula directly: rank and transition probability based measures.

This paper is organised as follows. Section 2 introduces ranks, copulas, and the concordance order. Section 3 establishes that widely used mobility measures are monotone in concordance. Section 4 explains why measures disagree when economies are not comparable with concordance and discusses an axiomatic completion with a minimum-distance implementation. Section 5 relaxes the common marginals assumption and Section 6 microfounds concordance in a model of endogenous human capital formation. Section 7 revisits trends in intergenerational mobility, showing measures agree in practice in line with theory. Finally, Section 8 concludes.

1.1. Related Literature

There is a large body of work proposing and estimating empirically relevant mobility measures. Surveys of this extensive literature include Black and Devereux (2011), Blanden (2013), Cholli and Durlauf (2022), Corak (2013), Deutscher and Mazumder (2023), Fields and Ok (1999a), Jäntti and Jenkins (2015), Solon (2002), and Stuhler and Biagi (2018). This paper studies how different measures are interconnected.

I focus on the implications of primitives of the joint-distribution for the ordering of mobility measures. This is closely related to axiomatic approaches that construct classes of measures from transparent notions of minimal and maximal mobility (this includes Cowell and Flachaire 2018; Fields and Ok 1996, 1999b; Shorrocks 1978). In contrast, the approach in this paper reverses the line of inquiry by showing the common properties of many measures. In perhaps the most closely related studies of income mobility, D'Agostino and Dardanoni (2009) axiomatize concordance as a measure of rank mobility with discrete transition matrices. Additionally, for a given sample, D'Agostino and Dardanoni convert the ordinal concordance ordering into a cardinal measure by calculating the percentage of rank mobility obtained relative to a theoretical maximum. In other closely related work, Atkinson and Bourguignon (1982) consider how dependence affects social welfare in the presence of multi-dimensional inequality. Their restrictions on dependence are mathematically similar to the concordance order. I study a general setting and relate to mobility measures beyond rank mobility or the impliciations for utilitarian social welfare functions.

I focus on the properties and correlational structure of estimands of intergenerational mobility rather than their estimators. A related strand of the literature considers inference and the role of measurement error for rank-rank specifications (e.g., Chetverikov and Wilhelm 2023; Kitagawa, Nybom, and Stuhler 2018), identification under general family tree structures (e.g., Collado, Ortuño-Ortín, and Stuhler 2023; Espín-Sánchez, Ferrie, and Vickers 2023), and biases in standard estimators (e.g., Haider and Solon 2006; Nybom and Stuhler 2017).

I focus on a single outcome for each generation. Instead, researchers may observe a vector of incomes over the life cycle (Mello, Nybom, and Stuhler 2022; Boserup, Kopczuk, and Kreiner 2018) or outcomes for multiple generations (Adermon, Lindahl, and Palme 2021; Ward, Buckles, and Price 2025) or multiple parents as inputs (Althoff, Gray, and Reichardt 2025).[2] Orthant orders (Shaked and Shanthikumar 2007) or other multivariate extensions (Joe 1997) generalize the concordance order to multivariate settings. Audoly et al. (2024) provide an alternative approach to summarizing multivariate joint distributions using heirarchical agglomerative clustering to study 25-dimensional outcomes.

This paper focuses on joint-distributions and is silent on measures of absolute mobility. These include panel independent measures of absolute mobility such as the fraction of children who exceed the median income in the parent generation (Katz and Krueger 2017). In an important recent contribution, Ray and Genicot (2023) show that panel independent mobility measures can be constructed if welfare does not depend on origin and destination identities over-and-above the progressivity of growth rates. These mobility measures capture planner preferences over the distribution of upwards absolute mobility.

Finally, this paper is related to a large body of work studying sorting and dependence properties. The main exercise in this paper is to show that the statistical property of concordance, which is directly verifiable from the joint density of ranks, implies a common ordering over a range of mobility measures widely used in empirical research. Under the concordance order, these measures are sufficient statistics for each other. Similarly to the analysis here, McGee (2023) studies the implications of increased concordance for adverse selection. Gola (2021), Anderson and Smith (2024) and Boerma et al. (2023) derive concordance orderings as the endogenous outcome of assignment problems and use it to measure sorting in the labour market. Like these studies, I show that increased concordance naturally arises as the outcome of endogenous choices. While they focus on sorting, for example in the labour market or between spouses, I show that the concordance order can be

[2] Blume et al. (2024) take an alternative approach to the multigenerational problem and measure mobility using the memory across generations induced by the markov chain

motivated by comparative statics in a Becker and Tomes (1979) style framework for endogenous human capital investment and intergenerational mobility.

Chiappori et al. (2025) axiomatise an odds-ratio index for studying assortative mating that is both necessary and sufficient for concordance in discrete contingency tables. Consequently, the axiomatic underpinnings of positive assortative mating measures provide a natural, alternative justification for employing the concordance ordering (and their completions) in mobility settings. The connections extend to other measures studied in this paper, for instance, the normalized-trace index in Chiappori et al. (2025) is a direct transformation of the Shorrocks (1978) mobility measure.

2. Setting and Concordance Ordering

This paper considers the following setting. A researcher is comparing economic mobility between economies. Specifically, the focus is on the joint behavior of parent–child (or period-to-period) outcomes and how that dependence shapes measures of mobility. I begin by defining the relevant joint distributions and providing the definition of the concordance order.

Let (Y^K, Y^P), indexed for child and parent respectively, be described by a joint distribution with CDF $F(Y^K, Y^P)$, and marginals $F_K(Y^K)$ and $F_P(Y^P)$. Define ranks in their respective distribution as $R^K = F_K(Y^K)$ and $R^P = F_P(Y^P)$, and the joint distribution of ranks by the *copula* of F, $C(R^K, R^P)$.[3]

By Sklar's Theorem (1959)

$$F(Y^K, Y^P) = C(F_K(Y^K), F_P(Y^P)), \tag{1}$$

[3] I treat the copula and marginals as observed. Alternatively, one can consistently estimate the relevant moments and I abstract from inference which can be non-trivial for rank-based measures (see Chetverikov and Wilhelm 2023). Appendix A discusses uniqueness of the copula and provides regulatory conditions for discrete cases.

so that the copula C alone encodes the entire rank-dependence between parent and child outcomes, independently of the marginals. This does not require assuming a specific functional form for the joint distribution and separates assumptions on *dependence* properties from those on marginal distributions.

The key intuition behind the results in this paper is that mobility measures hinge on the copula because it fully characterizes relative positions; any single-statistic index of relative mobility is simply a summary of that copula.[4]

To isolate dependence, the majority of the analysis will fix marginal distributions and focus solely on changes in the copula. For instance, imagine a thought experiment in which the U.S. marginal income distributions remain constant, but the parent–child rank-dependence structure varies. Under this restriction, each mobility measure becomes an alternative summary of the same copula—that is, of how often "high" outcomes for one generation coincide with "high" outcomes for the next. Section 5 shows how relaxing the constant-marginals assumption affects the results.

2.1. The Concordance Order

The central result in this paper is that concordance, a dependence property directly verifiable from the copula, implies a common ordering over many mobility measures widely used in empirical research.

Figure 1 presents a grid of parent–child income-quintile transition matrices, each illustrating a stepwise increase in concordance.[5] The top-left panel shows perfect discordance, children's ranks invert those of their parents, while the center panel depicts complete independence, with uniform conditional distributions. The bottom-right panel exhibits perfect concordance where children's ranks match parents'

[4] In addition, this property is exploited by Chetty et al. (2017), Berman (2022), and Manduca et al. (2024) who impose copula restrictions to bound absolute mobility when panel data are unavailable. Copulas also accommodate mass points—for example, incomes truncated by top- or bottom-coding or zero incomes from non-participation in the labour force.

[5] These examples assume outcomes take on discrete values for simplicity.

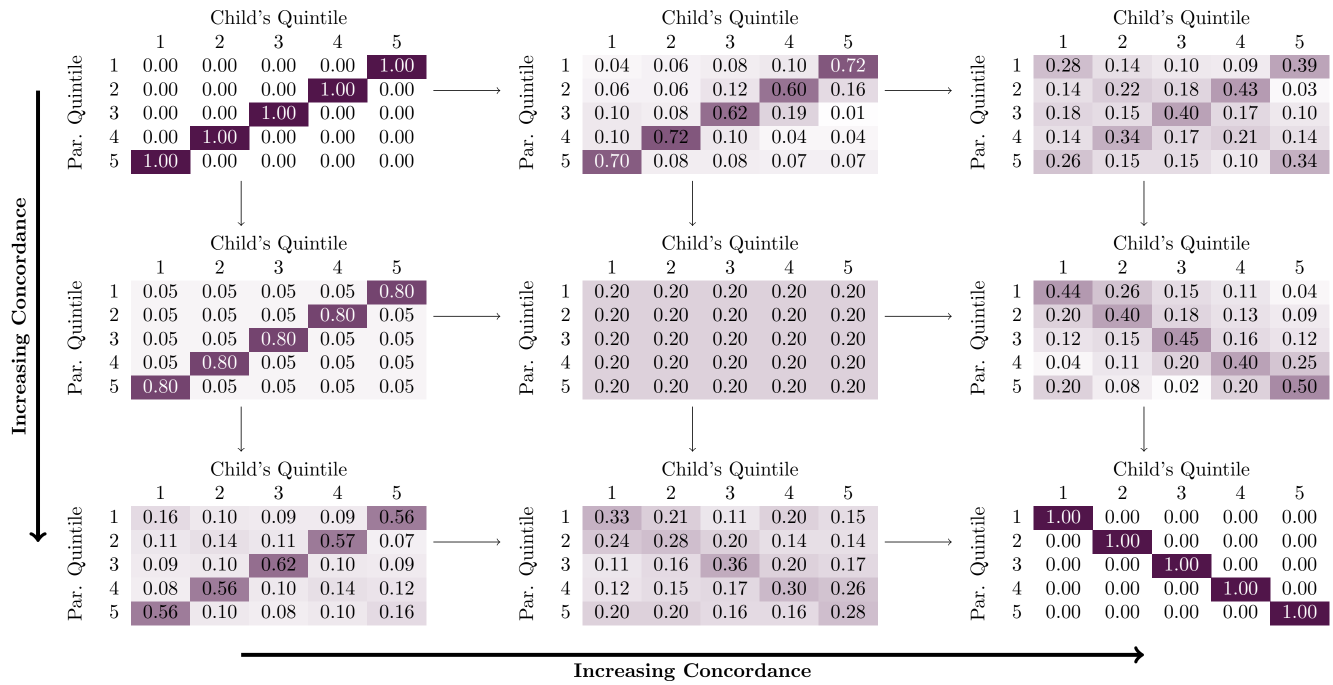


FIGURE 1. Concordance Ordering Among Transition Matrices

exactly. Moving rightward across any row or downward across any column is associated with increased concordance: probability mass shifts from the anti-diagonal toward the principal diagonal, tightening the dependence.

In the middle entry of the top row, the sum of the anti-diagonal entries is still large, but the conditional probability distribution is more diffuse relative to the top-left panel. In the final column of the top row, more mass shifts to the principal diagonal. For those with parent's outside of the middle quintile, the conditional distribution is now bi-modal. Beginning with this final transition matrix and moving down the column, we see the same increased concentration of mass towards the diagonal of the transition matrix, culminating in the extreme case in the bottom-right corner.

The statistical property of concordance formalizes the intuitive idea that "large" values of one random variable tend to coincide with "large" values of another: a form of stochastic dominance for dependence in joint-distributions (e.g., Kirkegaard 2017). In the context of intergenerational mobility copulas, it is higher if parental ranks are more likely to be realised with higher child ranks. Importantly, it appeals to an implicit sense of mobility. For example, the bounding exercise in Chetty et al. (2017) assumes the copula satisfy the "*intuitive requirement that children from higher-income families are less likely to have lower incomes*" (pg. 401). Concordance formalises these types of intuitions in a mathematically precise and testable stochastic ordering over copulas:[6]

Definition 1 (Concordance Ordering). *Let C^A denote the copula, or joint distribution of ranks, in economy A and C^B denote the copula in economy B. The dependence between outcomes Y^K, and Y^P is said to be larger in concordance in economy A than economy B (Yanagimoto and Okamoto 1969; Tchen 1980) if and only if $C^A(u,v) \geq C^B(u,v)\ \forall\, u, v \in [0,1]$.*

This is denoted $C^A \succeq C^B$ or, in a slight abuse of notation, $A \succeq B$.

This definition formalizes increased dependence between Y^K and Y^P in economy A relative to economy B. Statistical tests of concordance are provided in Cebrián, Denuit, and Scaillet (2004) and Denuit and Scaillet (2004) or can be constructed from moment-inequality testing procedures (Andrews and Shi 2013).

Formally, the concordance order captures two distinct concepts. First, the degree of monotone dependence or the tendency of random variables to cluster around the graph of any (measurable) function $Y^K = f(Y^P)$ or $Y^P = g(Y^K)$. Second, the direction of monotonicity—whether the function is monotone increasing or decreasing. Consequently, when viewed as a measure of mobility, this orders economies from most to least mobile. In the most mobile economies parent ranks fully determine child ranks via a decreasing function (perfectly reversing positions), akin to maximal negative rank correlation and generalising Prais (1955)'s origin independence. In contrast, in the least mobile societies parent ranks fully determine child ranks via an increasing function (perfectly maintaining positions), akin to maximal positive rank correlation.

[6] Throughout I will use superscript A and B to denote primitives and measures in different economies A and B, respectively. This ordering is sometimes alternatively known as the PQD ordering or point-wise copula ordering.

The following elementary exchanges define a sequence of copula transformations ordering joint-densities by their concordance (see also Tchen 1980):

Definition 2 (Concordance Increasing Exchanges). *Let C^A be a copula. Pick $0 < u_1 < u_2 < 1$ and $0 < v_1 < v_2 < 1$, and let $\varepsilon > 0$ be small enough that the update below remains a valid copula. Define*

$$C^B(u,v) \;=\; C^A(u,v) + \varepsilon\Big(\mathbf{1}_{\{u\geq u_1,\ v\geq v_1\}} + \mathbf{1}_{\{u\geq u_2,\ v\geq v_2\}} - \mathbf{1}_{\{u\geq u_1,\ v\geq v_2\}} - \mathbf{1}_{\{u\geq u_2,\ v\geq v_1\}}\Big).$$

We say C^B is obtained from C^A by a concordance-increasing exchange *on the rectangle $[u_1,u_2]\times[v_1,v_2]$. This operation preserves the marginals, $C^B(u,1) = u$ and $C^B(1,v) = v$, and moves probability mass from the two off-diagonal corners of the rectangle to the two diagonal corners. The reverse update with $-\varepsilon$ is concordance-decreasing.*

Note that with $u_2 - u_1 < \eta$ and $v_2 - v_1 < \eta$, for $\eta > 0$, this can be confined to arbitrarily small neighbourhoods. This provides an intuitive building block for increased concordance. For finite economies of size N, the equivalent definition is particularly intuitive. Let $(Y^K, Y^P)^A$ denote the pairs of incomes in finite economy A. The permutation $(Y^K, Y^P)^B$ is obtained from A by exchanging Y^K_i and Y^K_j (equivalently Y^P_i and Y^P_j) and leaving all other pairs unchanged. Then dependence between outcomes Y^K and Y^P is said to be larger in concordance in economy A than economy B if and only if $(Y^K_i - K^P_j)(Y^P_i - Y^P_j) < 0$.[7] Moreover, in both continuous and finite cases these exchanges can be applied iteratively and any increase in concordance can be expressed as a sequence of these elementary exchanges.

[7] Atkinson and Bourguignon (1982) highlight precisely these switches in the context of multi-dimensional inequality. This connects the results on measuring mobility across generations to measuring social welfare. When social welfare is utilitarian and sub-modular at the level of parent-child incomes, increased concordance lowers both mobility and social welfare.

Parametric Families of Distributions. While the copula approach does not restrict attention to a specific family of joint distributions, many commonly used families of joint distributions impose a concordance order as dependence increases (see Joe 2014, for a taxonomy). For instance, in a bivariate (log-)normal, raising the correlation parameter makes the two variables co-move more at all parts of the distribution. Thus, concordance also increases. Elliptical families more broadly, including those with heavy tails, behave this way as well. Similarly, multivariate extensions of the Singh and Maddala (1976) size-distribution for incomes, generalizing Champernowne (1952); Fisk (1961) and Pareto distributions, increase in concordance as the dependence parameter increases. More generally, the same monotone relationship shows up in many workhorse copula families (e.g., Clayton, Gumbel, Frank): increasing the single parameter controlling dependence shifts probability toward like-with-like (high–high and low–low) outcomes.

Stronger Dependence. The majority of the results in this paper follow directly from the concordance ordering. However, two of the results require an an empirically plausible and stronger regularity assumption on differences in the dependence structure:[8]

ASSUMPTION 1 (Anti-Diagonal to Diagonal Exchange in Mass). *Let* $\Delta C(u, v) \equiv C^A(u, v) - C^B(u, v)$ *denote the difference in the copulas A and B.* $\Delta C(u, v)$ *is a supermodular function: for all* $u_1 \leq u_2, v_1 \leq v_2 \in [0, 1]$

$$\Delta C(u_1, v_1) + \Delta C(u_2, v_2) \geq \Delta C(u_2, v_1) + \Delta C(u_1, v_2).$$

Intuitively, when moving from B to A similar ranks co-occur more often (low–low and high–high) and dissimilar ranks less often (low–high and high–low). When Assumption 1 is satisfied, this change occurs everywhere in the unit square, not just on average.

[8]The need to impose some restrictions on the joint density is a common problem in mobility research. As discussed above, versions of this empirical plausibility claim are invoked by Chetty et al. (2017) and Berman (2022). Shorrocks (1978) assumes away "*empirically unlikely*" transition matrices in the form of those without quasi-maximal diagonals.

Mass in the joint-distribution is pulled towards a central ridge along the diagonal, but does not create new ridges or foothills. It implies all sub-copulas satisfy both increased concordance and Assumption 1. For many parametric families of copulas, increasing dependence implies Assumption 1 is directly satisfied. The following lemma establishes how this new regularity condition is sufficient for concordance.

LEMMA 1 (Stronger Dependence). *All Copula pairs A and B that satisfy Assumption 1 also satisfy the concordance order in Definition 1.*

3. Concordance Ordering As A Sufficient Statistic For Exchange Mobility

I now show that the concordance ordering determines the rankings generated by a broad range of mobility metrics used in empirical research; including relative and weakly absolute indices, local and global measures, parametric and nonparametric estimands, and versions based on (fixed-)ranks, logs, or levels of income. Building on the taxonomy of Deutscher and Mazumder (2023) and axiomatic contributions such as Shorrocks (1978), I focus on exchange mobility measures that depend solely on the joint-distribution. For each metric, I prove that a rise in concordance induces a monotonic decrease in the mobility measure, producing a common ranking from most to least mobile (despite differing units).[9] In Section 6, I show that these measures also increase with investment. These coherence results are presented sequentially; formal proofs appear in Appendix B.

3.1. Regression

I begin by analysing two of the most widely used measures: the intergenerational earnings elasticity (IGE) and rank-rank regression slope. The IGE is the value of β,

[9] This echoes the "coherence" axiom of Scarsini (1984).

obtained from the following regression

$$\ln Y_i^K = \alpha + \beta \ln Y_i^P + \epsilon_i, \tag{2}$$

and the Rank-Rank regression slope, the value of ρ obtained from the following regression

$$R_i^K = \alpha + \rho R_i^P + \epsilon_i. \tag{3}$$

It is well known that the concordance order provides an ordering over linear regression coefficients. Thus, in the context of economic mobility, these mobility measures are unambiguously smaller in more concordant economies.

PROPOSITION 1 (Linear Regression Measures). *Economies A and B have identical marginals, but different rank dependence denoted by copulas C^A and C^B, respectively. If $C^A \succeq C^B$, then*

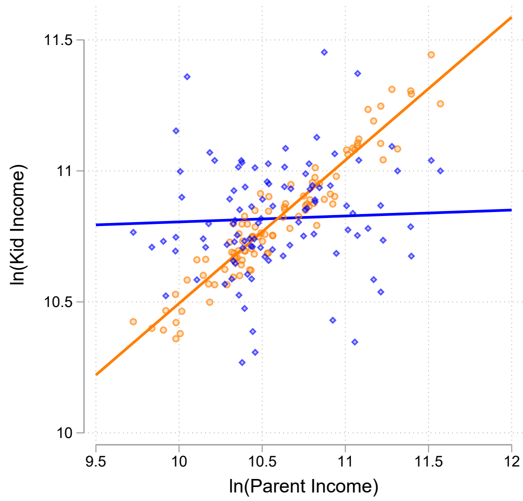
i. *The Intergenerational Earnings Elasticity is ordered: $\beta^A \geq \beta^B$; and*

ii. *The Rank-Rank Slope is ordered: $\rho^A \geq \rho^B$.*

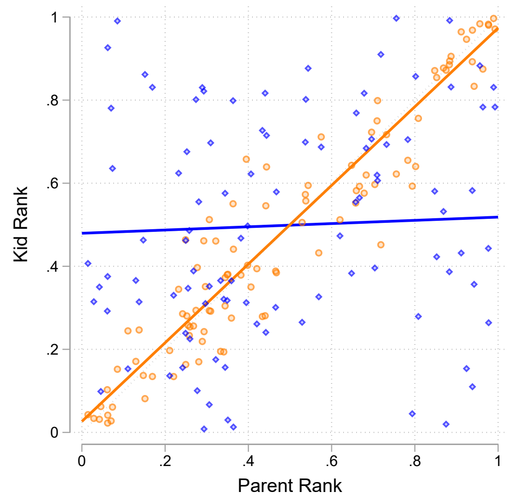
Both increase relative to the concordance ordering.

This establishes that both the intergenerational earnings elasticity and the rank-rank regression slope increase when concordance increases. Figure 2 displays the effect of increased concordance, as concordance increases from blue to orange distributions realisations in log- and rank-space are more tightly concentrated around co-monotonicity, and shows the result of Proposition 1 visually. A direct corollary is an order over correlation measures: the intergenerational correlation, $corr(\ln Y^K, \ln Y^P)$, and the rank correlation, $corr(R^K, R^P)$ also increase relative to the concordance ordering.[10]

[10] Hart (1983) and Shorrocks (1993) propose using one minus the intergenerational correlation as a measure of mobility. Similarly, increased concordance also raises *average* non-linear persistence measures (e.g., Arellano, Blundell, and Bonhomme 2017 or De Nardi, Fella, and Paz-Pardo 2020).



(A) Intergenerational Elasticity



(B) Rank-Rank Regressions

FIGURE 2. Regression Measures Under Increased Concordance

Notes: Each panel shows the effect of an increase in concordance on a mobility measure. The marginal distribution of parents income and children's income are held fixed. In each panel, the correlation parameter in the elliptical copula increase and the joint-distribution of the orange circle markers is larger than the blue diamonds in the concordance order. Both panels show the same 100 draws from the joint distribution and the population value of the mobility measure. Results correspond to Proposition 1, with the log-log regression (IGE) in panel (A) and the rank-rank correlation in panel (B).

In addition, both Proposition 1 and its corollaries can be extended to consider mobility for subsets of the population. For example, children born to parents in a specific segment of the income distribution. In these cases, the assumption of a concordance order can be relaxed to apply 'locally' in a segment of the income distribution.

Local Measures. Concordance orders standard regression estimates, but also the entire conditional linear relationship between generations and, therefore, local measures of mobility. Increased concordance rotates or tilts the entire non-parametric regression curve – lowering mobility measured by the conditional expected rank $E\left[R^K \mid R^P = r\right]$. I formalise this in the following proposition.

PROPOSITION 2 (Conditional Expected Rank Measures). *Economies A and B have identical* *marginals, but different rank dependence denoted by copulas C^A and C^B, respectively. For monotone conditional expected ranks, if $C^A \succeq C^B$ the conditional expected rank measure rotates around a point $r^\star$:*

i. It is decreasing relative to the concordance ordering below $r^\star$, $E^A\left[R^K \mid R^P = r\right] \leq E^B\left[R^K \mid R^P = r\right]\ \forall\, r \leq r^\star$, and

ii. It is increasing relative to the concordance ordering above $r^\star$, $E^A\left[R^K \mid R^P = r\right] \geq E^B\left[R^K \mid R^P = r\right]\ \forall\, r \geq r^\star$.

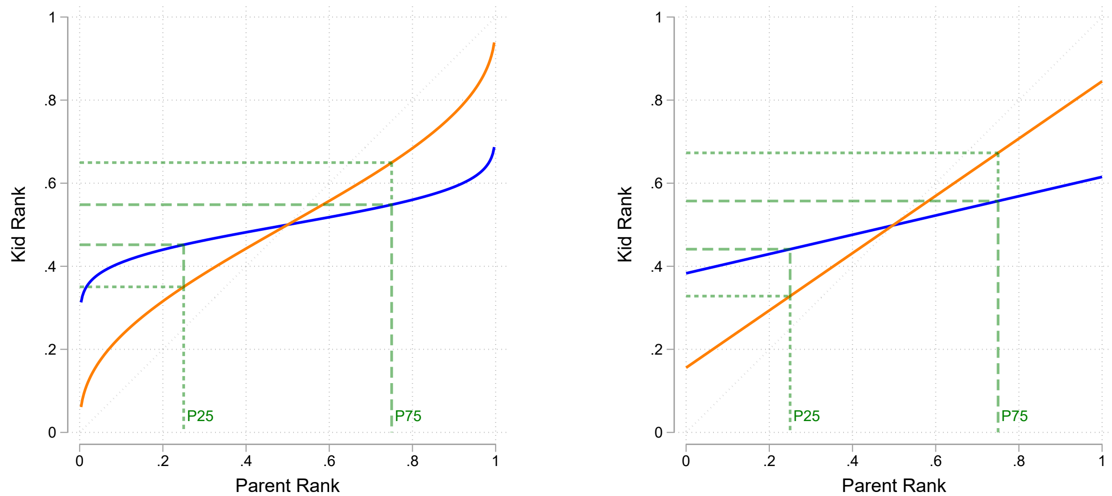
When the conditional expected rank is approximated parametricaly using the linear projection in equation (3), then $r^\star = 0.5$ and the conditional expected rank curve rotates around the median.

A key implication of higher concordance is a rotation of the nonparametric conditional expectation function, $E\left[R^K \mid R^P = r\right]$, toward the 45-degree line (Figure 3). Specifically, in more-concordant economies, the entire non-parametric conditional expected rank function becomes steeper. Children's expected rank responds more strongly to parental rank. Consequently, for low-rank parents their child's expected rank falls further below the median, and for high-rank parents, it rises further above. At the extreme of perfect concordance $E\left[R^K \mid R^P = r\right]$ lies perfectly on the 45-degree line and the conditional distributions are degenerate. Children's ranks coincide exactly with their parents' and relative mobility vanishes.

3.2. Transition Probabilities

Another class of local mobility measures targets specific segments of the distribution—often the "rags-to-riches" or "riches-to-rags" transitions (e.g., the probability of moving from the bottom quintile to the top, and vice versa) rather than average outcomes.[11] Because concordance orders the entire joint-distribution, it

[11]See Corak and Heisz (1999); Chetty et al. (2014a) for examples of this approach.



(A) Non-Parametric

(B) Linear-Approximation

FIGURE 3. Conditional Expected Ranks Under Increased Concordance

Notes: Each panel shows the effect of an increase in concordance on a mobility measure. The marginal distribution of parents income and children's income are held fixed. In each panel, the correlation parameter in the elliptical copula increase and the joint-distribution of the orange distribution is larger than the blue distribution in the concordance order. Both panels show how the population value of the mobility measure rotates corresponding to Proposition 2. Green lines correspond to the measure evaluated at the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentile— commonly used in the literature.

immediately ranks these transition probabilities: any increase in concordance raises the chance of staying near the diagonal (reducing extreme moves) and lowers the probability of crossing from one tail to the other.

PROPOSITION 3 (Rank Based Local Measures). *Economies A and B have identical marginals, but different rank dependence denoted by copulas C^A and C^B, respectively. If $C^A \succeq C^B$, then*

i. *Transition Probabilities: Measures along the positive diagonal are increasing relative to the concordance ordering, while those in off diagonals are decreasing:*

   a. *Positive diagonal:* $TP^A\left[R^K > \tau^k \mid R^P > \tau^p\right] \geq TP^B\left[R^K > \tau^k \mid R^P > \tau^p\right]$ *and* $TP^A\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right] \geq TP^B\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right]$ $\forall \tau^k, \tau^p \in [0, 1]$.

b. *Off-diagonal:* $TP^A\left[R^K > \tau^k \mid R^P \leq \tau^p\right] \leq TP^B\left[R^K > \tau^k \mid R^P \leq \tau^p\right]$ *and* $TP^A\left[R^K \leq \tau^k \mid R^P > \tau^p\right] \leq TP^B\left[R^K \leq \tau^k \mid R^P > \tau^p\right] \;\forall \tau^k, \tau^p \in [0,1].$

ii. *Bhattacharya and Mazumder (2011)'s Directional Rank Mobility: If Assumption 1 holds then the probability a child's rank is larger than their parent's rank, by an amount s, for those with parents below* τ *is decreasing relative to the concordance ordering:*

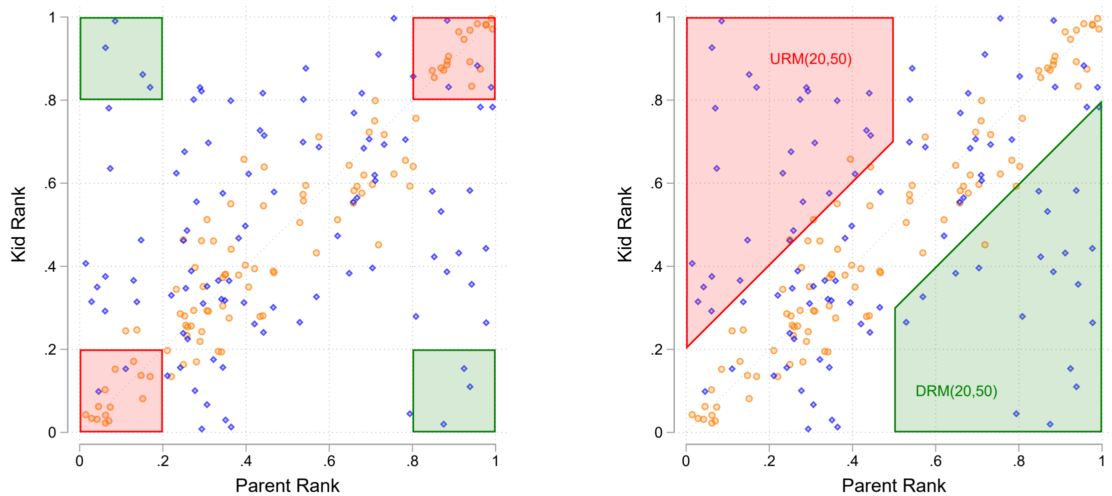
$$URM^B(s,\tau) \geq URM^A(s,\tau)\,\forall (s,\ \tau) \in [0,1]^2 \text{ where } URM(s,\tau) = Pr(R^K - R^P > s \mid R^P \leq \tau)$$

Analogously, Downward Rank Mobility (DRM).

Extreme transitions (e.g. bottom to top quintile) grow rarer, and the probability that a child surpasses their parent falls. Figure 4 illustrates this result visually, projecting probabilities in rank-space for specific threshold values, the frequency of these transitions declines with increased concordance. This highlights the power of concordance as a single dependence metric: across a range of local and global mobility notions, higher concordance uniformly implies lower mobility. Moreover, concordance ensures these results are not sensitive to the coarseness of the discretization or size of jumps.

3.3. Fixed Rank Comparisons

Empirical studies often evaluate ranks against a fixed external benchmark rather than within their own population. Examples include ranking children by their position in the parent cohort's income distribution; comparing subnational mobility against the national income distribution (e.g. Chetty et al. 2014a; Corak 2020; Bütikofer, Dalla-Zuanna, and Salvanes 2022; Deutscher and Mazumder 2020; Bell, Blundell, and Machin 2023); or for comparisons of outcomes across racial groups (in the spirit of Bayer and



(A) Transition Probabilities

(B) Directional Rank-Mobility

FIGURE 4. Rank-Space Probalities Under Increased Concordance

Notes: Each panel shows the effect of an increase in concordance on a mobility measure. The marginal distribution of parents income and children's income are held fixed. In each panel, the correlation parameter in the elliptical copula increase and the joint-distribution of the orange circle markers is larger than the blue diamonds in the concordance order. Both panels show 100 draws from the joint distribution and the population value of the mobility measure. Results correspond to Proposition 3. Panel (A) shows the off-diagonal (in green) and on-diagonal (in red) measures as regions of the rank-space using quintiles of the distribution. Panel (B) shows downwards rank mobility (in red) and upwards rank mobility as regions of the rank-space using above and below median parents who have children jumping more than a quintile in the child distribution.

Charles 2018).[12] Let

$$\widetilde{R}^K = F_{\widetilde{Y}}\left(F_{Y^K}^{-1}\left(R^K\right)\right), \tag{4}$$

be the child's rank in a fixed reference distribution $\widetilde{Y}$, e.g. their ranking in the parent's income distribution. Since this is strictly increasing in R^K it preserves the copula. Consequently, all rank-based exchange mobility measures remain monotone in concordance even under fixed benchmark rankings (an analogous argument applies for parent ranks). Those who with higher income ranks in their own distribution are also higher in the fixed reference distribution of incomes.

[12]Nonparametric estimation of these fixed-benchmark ranks corresponds to absolute mobility measures (Deutscher and Mazumder 2023).

3.4. Axiomatic Measures of Exchange Mobility

A complementary approach axiomatizes mobility measures (similarly to inequality measurement). They precisely define minimal and maximal mobility as well as necessary properties of mobility measures. Axiomatic frameworks (e.g. Fields and Ok 1996; Cowell and Flachaire 2018) specify distance metrics on parent–child distributions that uniquely define a mobility index. Likewise, D'Agostino and Dardanoni (2009) derive a rank-based concordance index from ordering axioms, and Shorrocks (1978)'s trace measure captures the probability of escaping one's parental "class". Under common marginals each of these indices hinges solely on the underlying dependence structure. Therefore, they all increase in the concordance order.

Proposition 4 (Axiomatic Measures). *The following axiomatic measures of exchange mobility are decreasing in the concordance order:*

i. *Fields and Ok (1996, 1999b)'s measures of absolute differences $\int\int |Y^K - Y^P| f\left(Y^K, Y^P\right) dY^K dY^P$ and it's decomposition into transitions.*

ii. *D'Agostino and Dardanoni (2009)'s concordance measure of rank mobility for global matrices and the extension .*

iii. *Cowell and Flachaire (2018)'s measure of the distance between relative positions, $\frac{1}{\alpha(\alpha-1)}\int\int \left(\frac{Y^K}{\bar{Y}^K}\right)^{\alpha}\left(\frac{Y^P}{\bar{Y}^P}\right)^{1-\alpha} f\left(Y^K, Y^P\right) dY^K dY^P$, where α controls the relative weight on upwards movements compared to downwards movements.*

Additionally, if Assumption 1 holds then

iv. *Shorrocks (1978)'s trace measure, $\frac{q-trace(\Omega)}{q-1}$, for stochastic matrix Ω with elements $\Omega_{i,j} = Pr\left(\frac{j-1}{q} < R^K \leq \frac{j}{q} \mid \frac{i-1}{q} < R^P \leq \frac{i}{q}\right)$.*[13]

[13] This is obtained by discretizing ranks into q groups. For simplicity, I consider the case where q groups are of equal size, but this is not integral to the result.

Figure 5 shows how an increase in concordance effects these mobility measures. The distributions of absolute differences is more compressed (Panel A) and relative statuses (Panel B) fall on average, lowering Fields and Ok (1996) and Cowell and Flachaire (2018) measures. At the same time, the shift towards co-monotonicity raises the probability along the trace of the discretized transition matrix (Panel C).

3.5. Health, Education, Social Class or Other Proxies of Socio-Economic Status

The mobility measures above are all constructed using the outcome of interest such as income or its ranks, but apply equally to mobility in other outcome variables. While researchers may be directly interested in these outcomes, they may also be motivated by the availability of data (e.g., Neidhöfer, Serrano, and Gasparini 2018, who harmonise educational achievement for a range of Latin American countries). I show that the concordance order is maintained when using proxies.[14]

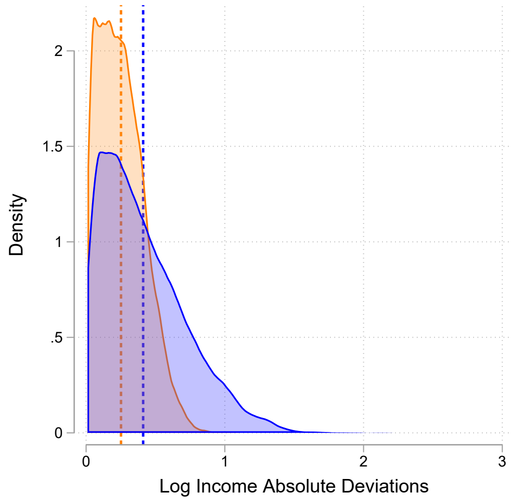
PROPOSITION 5 (Concordance of proxies). *Let a proxy E be determined by the following production function*

$$\begin{aligned} E^K =& H^K\left(R^K, \varepsilon^K\right) \\ E^P =& H^P\left(R^P, \varepsilon^P\right). \end{aligned}$$

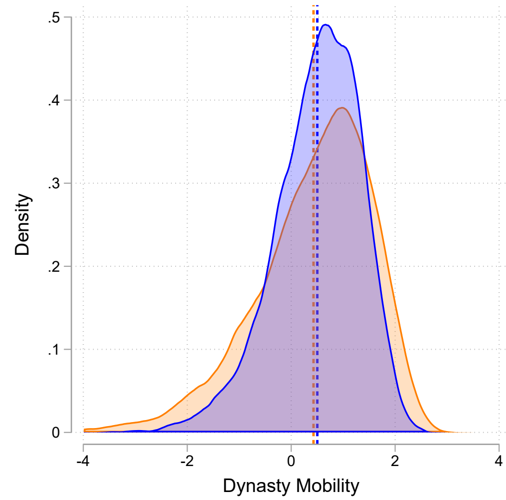
Assume $H^x(\cdot,\cdot)$ are weakly increasing function for $x \in \{K, P\}$ and $(\varepsilon^K, \varepsilon^P)$ is independent of the pair R^K, R^P with a joint-distribution that is the same in economy A and economy B. Then if $C^A \succeq C^B$ the pair $\widetilde{C}^A(E^K, E^P) \succeq \widetilde{C}^B(E^K, E^P)$.

Directly modeling dependence via copulas accommodates a variety of empirically relevant proxy frameworks, e.g. in education. In particular, it captures cases where education proxies income and where latent ability drives both education and income:

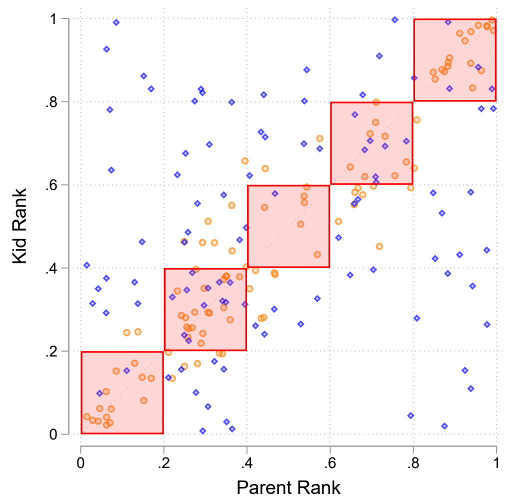
[14] Defining proxies as functions of ranks is without loss of generality under common marginals.



(A) Distribution of Fields and Ok (1996) Mobility



(B) Cowell and Flachaire (2018) Mobility



(C) Shorrocks (1978) Trace Measure

FIGURE 5. Axiomatic Measures Under Increased Concordance

Notes: Each panel shows the effect of an increase in concordance on a mobility measure. The marginal distribution of parents income and children's income are held fixed. In each panel, the correlation parameter in the elliptical copula increases and the joint-distribution of the orange distribution (circle markers) is larger than the blue joint-distribution (diamonds) in the concordance order. Panels (A) and (B) show the population distribution of individual dynasty mobility contributions as concordance changes along with mean values, corresponding to the value of the measure, in dashed vertical lines. Panel (C) shows the Shorrocks trace measure in red as regions of the rank-space using quintiles of the distribution. Panel (C) additionally shows 100 draws from the joint distribution. Results correspond to Proposition 4.

Example 1: Human capital determines incomes. Suppose increased human capital directly increases lifetime incomes (Mincer 1974) and we have $Y = H(E) + \varepsilon$. Inverting yields $E = H^{-1}(Y - \varepsilon)$. Treating ε as a vector allows for a general functional form for H and Y which can accommodate cases where the variance of the income residual increases in education as is standard in empirical work.

Example 2: Ability determines education and income. Alternatively, let a latent skill S determine both education and income: $E = H_E(S, \varepsilon)$ and $Y = H_Y(S, \varepsilon)$.

In each example the function H can differ across generations. Parent-child joint distributions, e.g. $F(Y^K, E^P)$ or $F(E^K, E^P)$, inherit the dependence structure of the copula $C(R^K, R^P)$, so concordance results extend immediately.

3.5.1. Measurement Error

Finally, I turn to understanding the impact of measurement error. Although noise attenuates dependence, any two economies ordered by concordance remain ordered once incomes or ranks are measured with error with the same measurement equations. Intuitively, miss-measured incomes act like proxies. The following corollary formalises this connection in the typical case where measurement errors are independent in each generation. Thus, even though estimates like rank–rank slopes or elasticities may be biased in level, their ordering across economies is invariant to measurement error.

COROLLARY 1 (Concordance under miss-measurement). *Let* $(\varepsilon^K, \varepsilon^P) \perp (R^K, R^P)$ *denote a vector of measurement errors that are independent of true incomes or ranks with* $\varepsilon^K \perp \varepsilon^P$. *Reported, or observed, incomes* $(\widetilde{Y}^K, \widetilde{Y}^P)$ *are increasing functions of true lifetime incomes (or ranks) and the measurement error. Therefore, they satisfy, the following measurement* *equations*

$$\widetilde{Y}^K = G_k(R^K, \varepsilon^K), \qquad \widetilde{Y}^P = G_p(R^P, \varepsilon^P),$$

for weakly increasing functions G_K and G_P.

Holding measurement equations and the distribution of measurement errors constant, if $C^A \succeq C^B$ then $\widetilde{C}^A \succeq \widetilde{C}^B$, where $\widetilde{C}(u, v)$ denotes the copula for the reported or observed outcomes $(\widetilde{Y}^K, \widetilde{Y}^P)$.

This result does not imply rankings across measurements are invariant to an increase in measurement error. Instead, adding the same measurement error to two economies preservers their ranking. The following three examples connect this result to empirically relevant settings.

Example 1: Classical measurement error in incomes. When observations are noisy measures of true lifetime income, this gives the following measurement equations satisfying the assumption in Corollary 1:

$$\begin{aligned}\widetilde{Y}^K &= F_K^{-1}(R^K) + \varepsilon^K \\ \widetilde{Y}^P &= F_P^{-1}(R^P) + \varepsilon^P.\end{aligned}$$

Example 2: Measurement error in ranks. When ranks are calculated on a noisy measure of true incomes (as in Chetverikov and Wilhelm 2023), this can be expressed as (satisfying Corollary 1):

$$\begin{aligned}\widetilde{Y}^K &= F_{\widetilde{K}}\left(F_K^{-1}(R^K) + \varepsilon^K\right) \\ \widetilde{Y}^P &= F_{\widetilde{P}}\left(F_P^{-1}(R^P) + \varepsilon^P\right).\end{aligned}$$

Example 3: Life Cycle Biases. Systematic variation in the link between current and life-time earnings generates bias in estimates of the intergenerational elasticity (Jenkins 1987; Nybom and Stuhler 2016). Kitagawa, Nybom, and Stuhler (2018) derive results for generalised measurement equations of the form

$$\widetilde{Y}^K = G_K\left(F_K^{-1}(R^K) + \mu_{\varepsilon^K} + \sigma_{\varepsilon^K}\varepsilon^K\right)$$
$$\widetilde{Y}^P = G_P\left(F_P^{-1}(R^P) + \mu_{\varepsilon^P} + \sigma_{\varepsilon^P}\varepsilon^P\right),$$

which also satisfy the restrictions in Corollary 1.

Thus, even if measurement error biases the construction of observed measures it does not affect the concordance order and, if true lifetime income ranks satisfy a concordance order, so too do miss-measured incomes or ranks. Similar points are made by Bhattacharya and Mazumder (2011) in the context of transition probabilities and Kitagawa, Nybom, and Stuhler (2018) in the context of rank-rank correlations. Importantly, assuming observed measures are monotone functions of true earnings does not rule out non-classical measurement error (in the spirit of Bound et al. 1994).

3.6. Summary

This section establishes that concordance is a strong measure of dependence. It delivers a coherent ranking across empirically relevant exchange mobility measures - whether based on regressions, transition-matrix summaries, axiomatic distances, or proxy-adjusted outcomes - and that these rankings remain intact even under measurement error. Concordance is a sufficient condition for all these mobility metrics to move in the same direction. Table 1 provides an overview of results, grouping mobility measures by methodological approach and summarizing the coherence properties and restrictions used to order all commonly used mobility measures.

TABLE 1. Properties of Exchange Mobility Measures & the Concordance Order

	Properties		Concordance Ordering				
	Copula only?	Distribution of what?	Increasing?	Mass points?	Fixed ranks?	Common marginals only?	Proxies or measurement error?
Regression measures							
IGE	No	Log income	Yes	Yes	N/A	Yes	Yes
Rank–rank correlation	Yes	Ranks	Yes	Yes	Yes	No	Yes
CER	Yes	Ranks	Yes[a]	Parametric Only	Yes	No	Yes
Transition matrix measures							
TP	Yes	Ranks	Yes	Yes	Yes	No	Yes
URM/DRM	Yes	Ranks	Yes	No	Yes	No	Yes
Axiomatic measures							
Shorrocks	Yes	Ranks	Yes[a]	Yes	Yes	No	Yes
Fields & Ok	No	Income levels	Yes	Yes	N/A	No[b]	Yes
D'Agostino & Dardanoni	Yes	Ranks	Yes	Yes	Yes	No	Yes
Cowell & Flachaire	User choice	Income levels or ranks	Yes	Yes	Yes	No[b]	Yes

Notes: [a] refers to results that additionally require Assumption 1. [b] denotes that common marginals are required unless the measure is evaluated using ranks or marginals also satisfy the usual stochastic order. Results that rule out atoms in column 4 also require continuous proxies or measurement error. Results summarize propositions in Sections 3 and 5. Proofs of the results are given in Appendix B.

Relation to Inequality Measurement. When dependence increases in the sense measured by the concordance order, mobility measures decrease. This has a natural analog in the study of inequality where second order stochastic dominance provides a partial order over economies and, in particular, their Gini coefficients and Lorenz curves (Atkinson et al. 1970).[15] Concordance is the natural extension of second order stochastic dominance to mobility: varying dependence while fixing marginal distributions, just as second order stochastic dominance varies inequality while holding means fixed. Exchanging individuals, as in Definition 2, form a similar role to the Pigou-Dalton principle in inequality measurement. This leads to rotations of the conditional expected rank curves (Figure 3) that parallel convexifying Lorenz curves.

4. Completing the Partial Order

The results established thus far show that the strong notion of dependence embedded in concordance is sufficient to order commonly used measures of economic mobility. While concordance offers an intuitive measure of mobility, it is only a partial order. Not all economies (or joint-densities) can be ranked by concordance.

Figure 6 illustrates this directly. Within each column, transition matrices increase in concordance, but no global ranking exists across columns or within rows. Berman (2022) argues copulas in the left-column are empirically plausible (comparing to estimates in Jantti et al. 2006, Eberharter 2013, and Chetty et al. 2017), but rejects the right-column "all-or-nothing" copulas with diagonal mass plus extreme jumps as implausible.

In each row, the two matrices in both yield the same rank correlation. In the first row, rank correlation is 0.32 and it is 0.65 in the second row. However, they do not produce equal values for all mobility measures. For example, in the first row, the matrices

[15] A similar connection was first proposed by Dardanoni (1993) who coins a *Dynamic* Pigou-Dalton principle, although earlier work connects elementary operations in Definition 2 to concepts of mobility (See Atkinson 1983, Ch. 3).

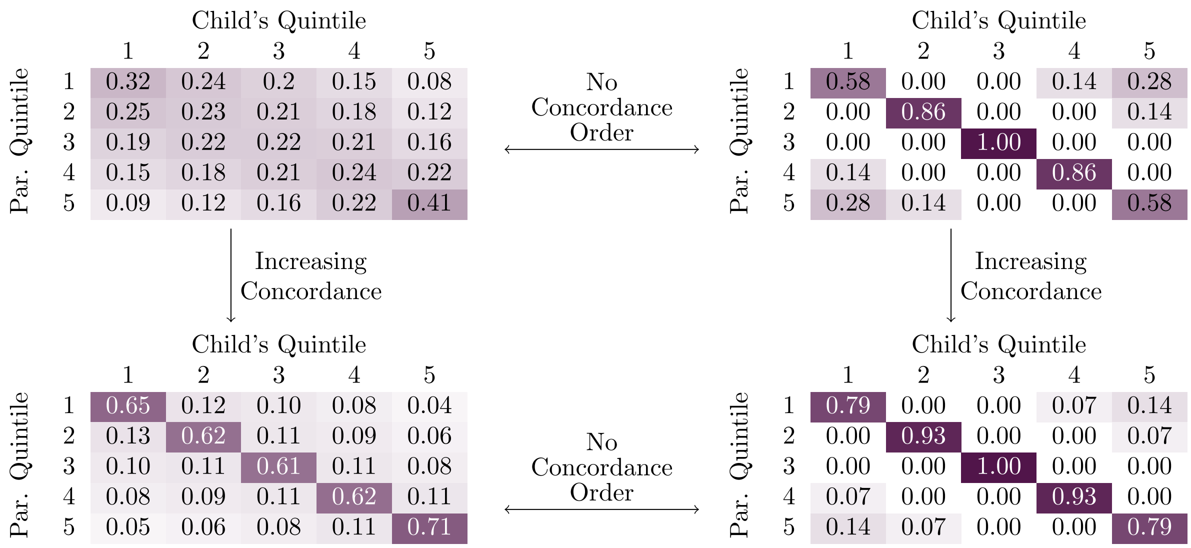


FIGURE 6. An Example of Concordance as a Partial Order

Notes: The first row is adapted from an example in Berman (2022). Each row produces numerically equivalent values for the rank correlation.

produce different probabilities of transitioning from the bottom quintile of parental income to the top quintile of child incomes.

This highlights an important point: different mobility measures place different weights on different parts of the joint distribution. Commonly used mobility measures can be expressed as weighted integrals of the joint density,

$$M_M = \int \int \phi^M \left(Y^K, Y^P\right) f \left(Y^K, Y^P\right) dY^K dY^P, \tag{5}$$

where $\phi^M\left(Y^K, Y^P\right)$ is a mobility measure specific intensity on parent-child income pairs. When comparing two different joint-distributions, different measures place different intensities on the change in the density (which must sum to zero) across economies. For example, the intergenerational elasticity emphasizes the changes in density using the covariance term, $\phi^M\left(Y^K, Y^P\right) = (Y^K - \overline{Y}^K)(Y^P - \overline{Y}^P)$, so that changes in the density at the extremes of the distribution matter more. The Cowell and

Flachaire (2018) intensities are proportional to a Cobb-Douglas aggregator of parent and child incomes, $\phi^M\left(Y^K, Y^P\right) \propto (Y^K/\overline{Y}^K)^{\alpha}(Y^P/\overline{Y}^P)^{1-\alpha}$. Depending on the degree of supermodularity, α, they emphasize extreme directional transitions more than those near the mean. In contrast, transition probability measures have intensities defined by indicator functions—changes in densities elsewhere in the distribution have no effect on the measures.[16]

When joint-distributions are ranked in their concordance, shifts in the density ensure mobility measures move in the same direction for all of these intensities. When two distributions are not comparable by concordance, they can produce different orderings over mobility measures. This is precisely because different measures emphasize different parts of the joint distribution as in examples constructed from Figure 6. When economies are not ordered by concordance, the choice of measure involves an implicit ethical judgment over what constitutes desirable mobility (Fields and Ok 1996; Jäntti and Jenkins 2015; Ray and Genicot 2023).

4.1. Completing the Concordance Order

There are two approaches to completing the concordance order. The first is to explicitly restrict attention to economies that can be ranked in their concordance (for example a single family of copulas). This directly rules out pathological shifts in the joint distribution in Figure 6 and retains only joint distributions identified as empirically implausible in the prior literature.

A second approach is to construct a specific tie-breaking rule. D'Agostino and Dardanoni (2009) axiomatize completions of the concordance order for discrete transition matrices (or contingency-tables).[17] Their axiomatic approach restricts the

[16]This is similar to requiring a fully specified social welfare function to weight changes in inequality when Lorenz curves intersect (Atkinson et al. 1970).

[17]See also Chiappori et al. (2025) in the mathematically equivalent context of positive assortative mating.

set of feasible mobility measures. To do so they require that: (i) within a family mobility increases as parent-child ranks are further apart in absolute distance, (ii) an Archimedean property that allows more (less) mobile societies to be bridged to by embedding them in less (more) mobile societies, and (iii) a minimal inversion criterion which requires one-position inversions have the same effect on mobility regardless of original location. This delivers a completion of the partial concordance order that is the sum of squared rank gaps or the Spearman-rank correlation in the continuous case.

The following intuitive minimum-distance estimator is (up to a positive scaling) identical to the complete order of D'Agostino and Dardanoni (2009) :

Definition 3 (A Minimum Distance Completion of the Concordance Order). *For any empirical copula $C(u, v)$, we can construct the following summary statistic, by mixing between the independent copula, $C^{\perp}_{uv} = uv$, and the Hoeffding upper bound, $M(u, v) = min(u, v)$,*

$$M_\lambda \in argmin_{\lambda \in [0,1]} \int_0^1 \int_0^1 \left[C(u, v) - ((1 - \lambda)\, M(u, v) + \lambda\, C^{\perp}_{uv}) \right]^2 du\, dv. \tag{6}$$

This increases in the concordance order and provides an axiomatic tie-breaking rule for rank mobility. Intuitively, the mobility measure approximates observed mobility with a mixture between origin independence and perfect predetermination. These mixtures decrease in concordance as the weight on origin independence increases. This weight provides the mobility index which is reported in the application below.[18] Fernández and Rogerson (2001) and Abbott et al. (2019) implement an identical minimum-distance estimator to (6) in studies of the marriage market.

[18]It is easy to show that the value of this mobility measure is given by $M_\lambda = 1 - \int_0^1 \int_0^1 \left(C(u, v) - u\, v\right) \left(\min(u, v) - u\, v\right) du\, dv \Big/ \int_0^1 \int_0^1 \left(\min(u, v) - u\, v\right)^2 du\, dv$.

5. Comparisons across time and place: relaxing common marginals

The results so far have assumed common marginals. This corresponds to comparing hypothetical variants of a single economy, but comparisons across space and time, such as asking whether U.S. is more mobile today than it was in 1965 (Chetty et al. 2017) or whether the U.S. is more mobile than Canada (Corak 2020), may not hold marginals fixed. I now ask whether the same results continue to hold.

As long as the copulas are ordered in concordance, then all rank based measures of mobility are ordered.[19] These results hold even without common marginal distributions of incomes because ranks themselves share a common (and uniform) marginal distribution. All rank based measures depend solely on the copula and not on the differing marginal distributions of income in levels. Proxy-based indices that map ranks (e.g., social-class categories) to observed proxies (e.g., occupations) are also ordered even when marginals differ across economies. In contrast, however, proxies that depend directly on the levels of income are not ordered.[20]

This robustness does not extend to level- or log-based mobility measures.[21] Consider an economy in which parent and child incomes are distributed log-normally with

$$\begin{pmatrix} \ln Y^K \\ \ln Y^P \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_P \\ \mu_K \end{pmatrix}, \begin{pmatrix} \sigma_K^2 & \rho \\ \rho & \sigma_P^2 \end{pmatrix}\right],$$

where ρ controls the dependence between generations and orders the underlying copula

[19] This implies rank based alternative versions of measures are ordered. For example, the Fields and Ok (1996) measure computed using ranks which corresponds to the Bartholomew (1973) average jump measure, a seminal contribution in quantitative sociology.

[20] The assumptions that proxies depend on ranks, and not levels, can be tested in auxiliary datasets without information on two generations.

[21] Under the stronger assumption of stochastic dominance of the marginal distributions, it is possible to obtain a general ranking for all super-modular measures (see Meyer and Strulovici 2013, for an example of this decomposition argument). This approach is particularly relevant for exercises that employ "copula and marginal" approximations to estimate absolute mobility.

in concordance. In this special case, the rank-rank correlation is given by $Corr(R^K, R^P) = {}^6/_\pi \arcsin(^\rho/_2) \approx \rho$ (Kruskal 1958) so that both the intergenerational correlation and the rank-rank correlation are almost identical. Thus, they have the same ordering over economies with different values of ρ. However, even in this case the ordering of the intergenerational elasticity, $\beta = \rho^{\sigma_K}/_{\sigma_P}$, can reverse even as ρ increases because it depends on changes in the marginal distributions of Y^K and Y^P through their relative dispersion as well as their dependence. More generally, differences in the shape and scale of incomes can tilt the measurement of mobility. Column 6 of Table 1 summarizes these limits, highlighting both where concordance ordering holds and where it does not.

6. The Concordance Order in a Model of Human Capital Investment

To give concordance an economic interpretation, I show how it arises in a modified version of the Becker and Tomes (1979) and Loury (1981) models of human capital investment and intergenerational mobility.[22]

A family consists of a parent and child indexed by P and K. Parents value their own consumption as well as their child's with altruism parameter $\delta = {}^1/_{1+r}$. They choose to consume, C, and invest, I, in the future human capital of their child. The human capital of their child depends on investment and the direct transmission of the parent's human capital, H^P, which are aggregated through a weakly-decreasing returns to scale production function f. It also depends on a term capturing luck, θ. They solve the

[22]Throughout I abstract from a number of margins that empirical work on early childhood development has shown to be important. These include the beliefs of parents (e.g., Attanasio, Cunha, and Jervis 2019; Attanasio, Boneva, and Rauh 2022), dynamic complementarity (e.g., Cunha and Heckman 2007; Caucutt and Lochner 2020), multiple dimensions of skill (e.g., Attanasio, Meghir, and Nix 2020), multiple dimensions of investment (e.g., time and money Del Boca, Flinn, and Wiswall 2014; Lee and Seshadri 2019; Attanasio et al. 2020; Mullins 2022, or multiple inputs Caucutt et al. 2020; Moschini 2023), and borrowing constraints (e.g., Caucutt and Lochner 2020; Bolt et al. 2025).

following maximisation problem:

$$\max_{C^P, I} U\left(C^P\right) + \delta \mathbb{E}_\theta\left[U\left(C^K\right)\right] \tag{7}$$

subject to

$$C^P = Y^P - I \tag{8}$$

$$C^K = Y^K = W(H^K) \tag{9}$$

$$H^K = \theta f(I, H^P) \tag{10}$$

When the production function is Cobb-Douglas and wages are linear in human capital this produces the standard intergenerational earnings elasticity specification. As in Loury (1981), the ability shock θ is realised after investments are made and is independent of familiy choices and state variables. Consequently, θ becomes the error-term in the reduced form intergenerational earnings elasticity specification.[23]

I show that concordance characterizes comparative statics in a stylized model of endogenous human capital investment by comparing how changes in model parameters shift the induced parent–child copula (therefore it shares a common microfoundation with the intergenerational elasticity, e.g. Solon 1992). I demonstrate that concordance is more than an abstract dependence restriction and, instead, reflects economically meaningful behavior.

For tractability, I additionally assume that the income of each child is an increasing function of their rank in the human capital distribution.[24] Given any equilibrium distribution of children's human capital this can be justified through a number of

[23]This is closer to a 'luck' interpretation of θ. In more recent work, e.g. Cunha, Heckman, and Schennach (2010); Caucutt and Lochner (2020), this θ is known at the time of investment and is the child's ability to learn or the parent's ability to teach. Under this alternative timing, the pointwise ordering of investment below instead holds for all (θ, Y^P) pairs and the expectation is degenerate. It is also possible to incorporate stochastic wages given human capital, although I omit this for parsimony. As Lochner and Park (2024) emphasize, in this case intergenerational income mobility will differ from intergenerational skill mobility.

[24]This is a small abuse of notation.

different microfoundations: a piece-rate in efficiency units; the assignment of heterogenous workers to heterogenous firms when production is supermodular (Becker 1973; Sattinger 1975; Rosen 1981; Shimer and Smith 2000); rank-order tournaments (Lazear and Rosen 1981); or a hierarchical job-assignment model (Costrell and Loury 2004). Importantly, it is analytically convenient to separate the role of sorting or intergenerational dependence from the marginal distribution of child incomes.[25] As it is ultimately the rank of the child's human capital that matters, this might reasonably be termed a model of economic status production.

I focus on increased concordance generated through the endogenous investment decision, which I formalise in the following lemma.

LEMMA 2 (Concordance and an outward expansion of investment). *Assume that the human capital production function $f(\cdot,\cdot)$ is weakly increasing and supermodular in both arguments. Then, holding the distribution of parental incomes fixed, any two economies A and B that produce the following pointwise relation on endogenous investment decisions $I^\star$,*

$$I^{\star A}\left(Y^P\right) \geq I^{\star B}\left(Y^P\right) \quad \forall\, Y^P, \tag{11}$$

also produce the following concordance order on parent-child incomes and their copulas:

$$A \succeq B \quad \textit{equivalently} \quad C^A \succeq C^B. \tag{12}$$

The following corollary links this directly to interpretable model primitives.[26]

[25] In comparative statics exercises this does not hold the marginal distribution of child incomes constant, but still allows economies to be ranked by the concordance of their copulas. It is trivial to see that this holds when we impose common marginals. More generally, this sets the model in a general equilibrium context (following Heckman, Lochner, and Taber 1998 or Lee and Wolpin 2006 for example) by allowing wages to adapt to changes in the supply and composition of educated workers without fully specifying general equilibrium forces.

[26] In Appendix C, I prove that a broad class of policy or parameter shifts, each raising the marginal return to investment, expand the investment policy function and tighten parent–child dependence.

Corollary 2 (Mechanisms producing increased concordance). *The following mechanisms all imply a pointwise order on investment policy functions,* $I^{\star A}\left(Y^P\right) \geq I^{\star B}\left(Y^P\right)$ $\forall Y^P$*, and, thus, a concordance order:*

i. *An increase in the marginal product of investment,* $f_H^A(I, H^P) \geq f_H^B(I, H^P)$ $\forall H^P, I$*, or returns to human capital,* $W^A(H^K) \geq W^B(H^K)$*;*

ii. *An investment subsidy or parental-income targeted investment intervention;*

iii. *An increase in the discount factor,* $\delta^A \geq \delta^B$.

In economic terms, amplifying the motive to invest increases the equilibrium contribution of investment while diluting the effect of luck. Thus, concordance is not just a statistical artifact, but is instead consistent with the optimizing behaviour of economic agents.

7. Revisiting Intergenerational Mobility Over the 20th Century

To link the theoretical results explored above to the study of intergenerational mobility, I construct cohort-specific mobility estimates for U.S. males born between 1910 and 1980. Following Jácome, Kuziemko, and Naidu (2025), I pool all available surveys that report respondents' current family income together with race, father's occupation, and region of birth or childhood and impute parental income from auxiliary data sources (primarily the Census) using race, education, and region.[27] This yields a repeated cross-section that is nationally representative and consistent over time. This allows me to document trends in intergenerational mobility measures over the 20th century (see also, Davis and Mazumder 2024).

[27] For linear estimating equations this is a Two-Sample Instrumental Variable design. Like many studies of mobility in a historical context (e.g., Collins and Wanamaker 2022; Ward 2023), this application uses both self-reported incomes and a proxy approach to construct linked parent-child outcomes. The theoretical charaterisation of mobility measures shows that results are robust to using proxies or the presence of measurement error.

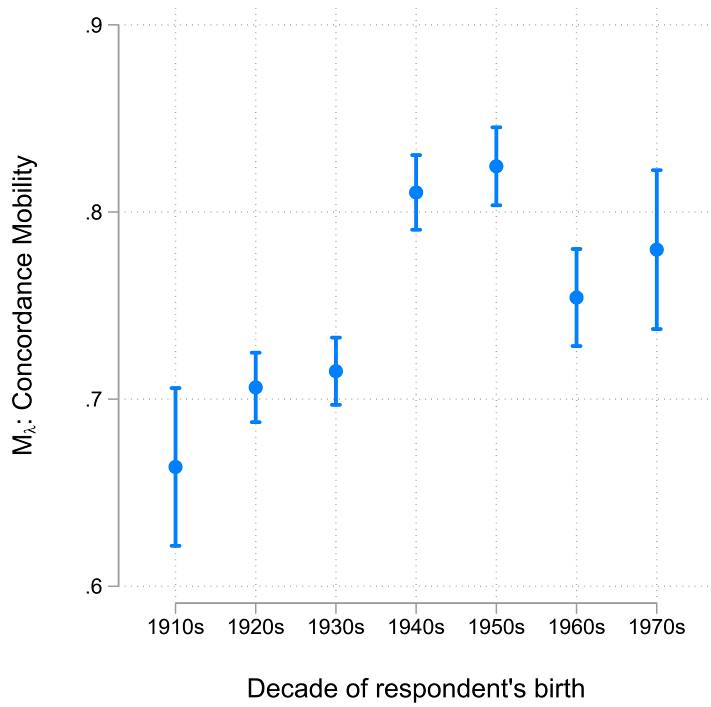


FIGURE 7. Concordance Mobility Over the 20$^{\text{th}}$ Century

Notes: Notes: Author's estimates from 15 combined US data sources (see text, Jácome, Kuziemko, and Naidu 2025 and Appendix D for details) by birth decade for respondents ages 30–50. Parental income predicted using family income conditional on father's occupation, race, and region (South vs. elsewhere) from auxiliary data (often the census) as close as possible to the respondents' tenth birthday (see Jácome, Kuziemko, and Naidu 2025 sec. III.B for more details). Sample weights are used and I reweight each birth cohort (i.e., decade) so that they have representative race$\times$sex shares. Estimate of concordance mobility measure in (6) and 95% confidence interval from 400 bootstrap replications.

I restrict the sample to fathers and sons aged between 30 and 50 (to limit life-cycle bias) and begin by documenting trends in the concordance measure of economic mobility using the completion of the partial order in equation (6). Figure 7 shows how the (dis-)concordance measure, the mixture-weight between co-monotone and independent copulas, evolves over the 20th century. Mobility increases from 0.66 to 0.78 over the 20th century. The point estimates are precisely estimated and the growth in intergenerational mobility is statistically significant. Notably, mobility has a wave-like pattern and does not rise monotonically. In line with the predictions of theory above, I show that this wave-like pattern is inherited by other summary measures of economic mobility.

I now turn to documenting trends in other intergenerational mobility measures in Figures 8 and 9: the intergenerational elasticity (IGE), the log-income correlation, and the rank-rank slope, along with directional and trace-based measures from the transition matrix, and axiomatic mobility measures. I normalize measures so that they increase with mobility (i.e., one minus persistence). Remaining details of the implementation are deferred to Appendix D.

Panel (A) of Figure 8 replicates the broad patterns in Jácome, Kuziemko, and Naidu (2025):[28] rank persistence and the income correlation fall, thus mobility rises, through the mid-20th century and then stabilise, while the IGE rises as income dispersion widens in the latter half of the century leading to a reversal of mobility. Turning to measures of directional mobility, Panel (B) shows that both Downward and Upward Rank Mobility (see Proposition 3 for definition) measures trend upwards over the 20th century.

Under the concordance order, the rank-rank mobility measure in Panel (A) is a sufficient statistic for all rank-based relative mobility measures. The first row of Figure 8 shows that this theoretical prediction is supported by the data. The wave-like pattern of decreasing rank-persistence (Panel A) is mirrored in the estimates of increasing directional mobility. Panel (C) shows the same trends hold true for mobility measures using the diagonal of the transition matrix. Both so-called "rags-to-rags" and "riches-to-riches"[29] exhibit a trend of declining persistence. Similarly Panel (C) also shows the Shorrocks measure tracks the same wave-like pattern of persistence. Finally, Panel (D) displays the analogous off-diagonal transitional probabilities.

Figure 9 repeats the exercise for two axiomatic indices. Panel (A) plots the Exchange

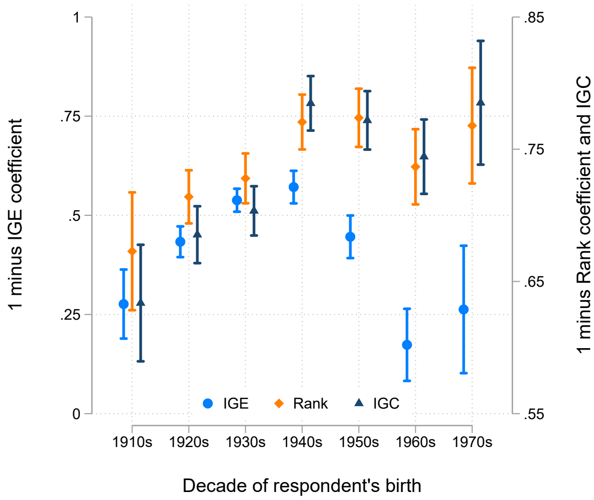
[28] This replicates results in their Figure 1. However, due to differences in the underlying datasets (see Appendix D) this is best thought of as a distinct replication rather than a reproduction. Results are qualitatively identical despite modest quantitative differences; lending support to the overall validity of their results.

[29] Remaining in the bottom and top quintile across generations, respectively.

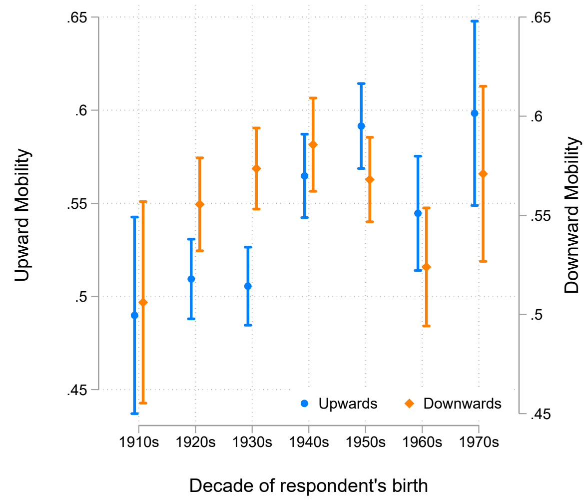
mobility measure of Fields and Ok; panel (B) shows the Cowell and Flachaire measure with "relative status" computed with income ranks and levels. Both rise across successive birth cohorts, echoing the upward trends in rank-based measures from Figure 8, even though the concordance order does not, by itself, determine the behaviour of these measures.

By contrast, the absolute version of the Fields and Ok index, which incorporates growth in mean income and changes in inequality, tells a different story. It climbs sharply for early-century cohorts but flattens thereafter, much like the reversal in the IGE. Aggregate income growth concentrated in the right tail of the income distribution (see, e.g., Blundell et al. 2018; Piketty, Saez, and Zucman 2018; Guvenen et al. 2022) offset the gains in relative mobility, leaving absolute mobility roughly constant in the latter half of the century.

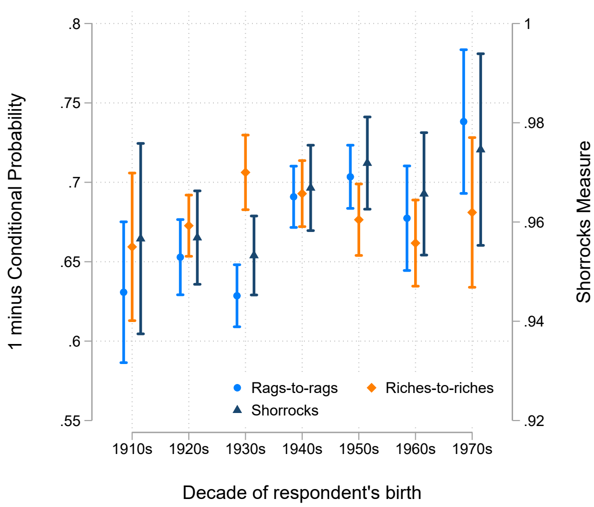
This shows that, despite imprecision in estimates of some mobility measures, results align closely with the theory. As the estimated rank-rank correlation and, therefore, the concordance falls across cohorts, the rank–rank slope falls, directional "rags-to-riches" and "riches-to-rags" probabilities increase, and the Shorrocks trace declines. This is exactly the pattern implied by higher concordance and the mobility measure from the completion of the concordance order. Absolute-mobility indices that incorporate mean growth and inequality rise early in the century but flatten thereafter, showing how shifting marginals can offset gains in relative mobility.



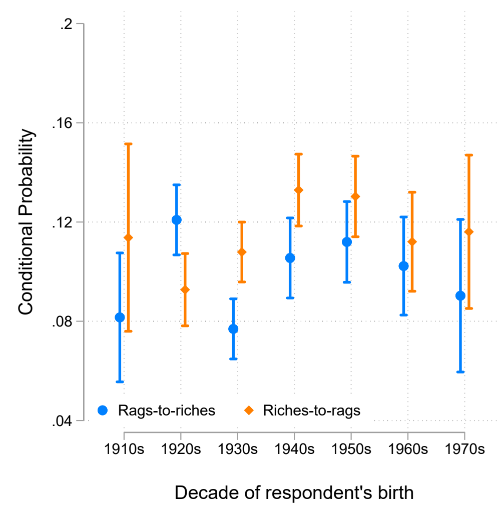
(A) Regression Estimates



(B) Directional Rank-Mobility



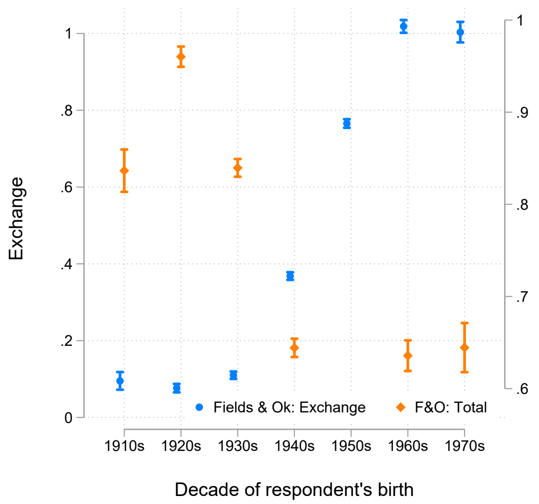
(C) Transition Probabilities (On-Diagonal)



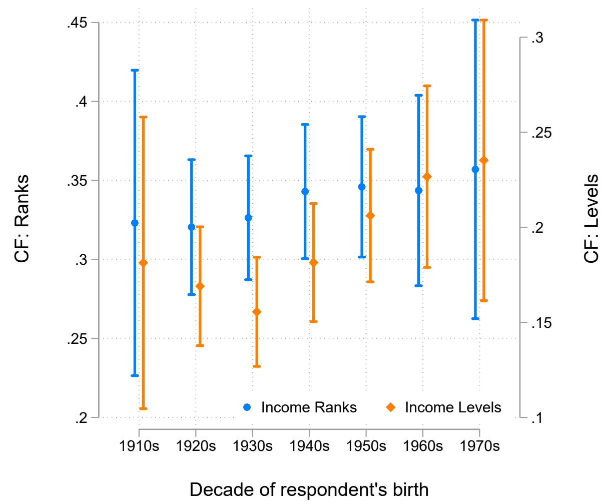
(D) Transition Probabilities (Off-Diagonal)

FIGURE 8. Trends in Intergenerational Mobility Measures

Notes: Author's estimates from 15 combined US data sources (see text, Jácome, Kuziemko, and Naidu 2025 and Appendix D for details). All measures by birth decade for respondents ages 30–50. Parental income predicted using family income conditional on father's occupation, race, and region (South vs. elsewhere) from auxiliary data (often the census) as close as possible to the respondents'tenth birthday (see Jácome, Kuziemko, and Naidu 2025 sec. III.B for more details). Sample weights are used and I reweight each birth cohort (i.e., decade) so that they have representative race×sex shares. Panel (A) reports estimates of IGE, IGC, and rank-rank slope corresponding to Proposition 1. Panel (B) reports estimates of directional rank mobility measures using quintiles ($Pr(R^K - R^P > 0.2 \mid R^P \leq 0.2)$ and $Pr(R^K - R^P < -0.2 \mid R^P > 0.8)$) in Proposition 3. Panels (C) and (D) report transition probabilities using quintiles (Proposition 3) and panel (C) additionally reports the Shorrocks Trace Measure (Proposition 4) using deciles. Panels (B)-(D) use 400 bootstrap replications for inference. Panels (A) and (C) normalise measures to increase with mobility (one minus persistence).



(A) Fields & Ok Measure



(B) Cowell & Flachaire Measure

FIGURE 9. Trends in Intergenerational Mobility Measures (cont.)

Notes: Author's estimates from 15 combined US data sources (see text, Jácome, Kuziemko, and Naidu 2025 and Appendix D for details). All measures by birth decade for respondents ages 30–50. Parental income predicted using family income conditional on father's occupation, race, and region (South vs. elsewhere) from auxiliary data (often the census) as close as possible to the respondents' tenth birthday (see Jácome, Kuziemko, and Naidu 2025 sec. III.B for more details). Sample weights are used and I reweight each birth cohort (i.e., decade) so that they have representative race×sex shares. Panels (A) and (B) report estimates of the Fields & Ok and Cowell & Flachaire measures (Proposition 4), respectively.

8. Discussion and Implications for Applied Researchers

The core insight in this paper is straightforward: when two economies or time periods can be ranked by concordance, any mobility measure—whether based on ranks, regressions, transition probabilities, or axiomatic distances—will agree on which setting is more or less mobile. In other words, immobility rises and mobility fall in tandem, across a wide array of exchange-mobility metrics if their underlying copulas are ordered by concordance.

This helps answer two important open question in the literature on intergenerational mobility (e.g., Berman 2022 and Deutscher and Mazumder 2023). First, when is one measure sufficient? If a collection of joint distributions is ordered by concordance, then any one metric serves as a sufficient statistic for all the others.

Second, which measure should the applied econometrician choose? Axiomatic arguments may not suffice, as under concordance all measures deliver the same ordering and thus the order shares a common axiomatic justification. Whenever two distributions cannot be compared under concordance, selecting a particular statistic amounts to choosing a tie-breaking rule for those incomparable cases. When they are ordered under concordance the choice of measures is, in a sense, redundant. Without an ex ante reason to prefer one measure to another a researcher may possess the ability to choose the ordering ex post (although inference on rankings is challenging, see Mogstad et al. 2024).

Even when different measures appear highly correlated in practice, concordance explains that agreement; when they diverge, it may reflect sampling variation or simply the partial nature of the concordance order. Outside of tie-breaking, there remain principled reasons to favor one metric over another. For example, different estimators can differ in their finite-sample efficiency or in the bias they incur under various forms of measurement error.

Some measures live in more familiar units—elasticities, correlation slopes or probabilities—which can aid interpretation—however, this may also allow researchers to manipulate notions of 'similar' mobility. Other measures may be better suited to descriptive or causal analysis of the underlying mechanisms. For example, they may be easier to decompose into the mobility of subgroups or be better suited to mediation analysis (as used in Fagereng, Mogstad, and Rønning 2021; Bolt et al. 2021). Finally, practical data considerations often drive the choice of measure: education proxies, for instance, are widely available even when lifetime-income ranks are not. Under concordance, those proxies still yield valid comparisons of relative positions.

The concordance order is primarily a statistical concept, but it also captures economically meaningful changes in behaviour. It naturally arises as a way to characterize comparative statics in a standard model of endogenous human capital formation. This provides a novel microfoundation for the abstract concept.

Concordance also makes it possible to knit together mobility rankings across datasets that use different measures. When economies are linked by a chain of pairwise concordance comparisons—so that each economy shares at least one mobility metric with another in the chain the ordering can be propagated throughout that connected group, even if no single measure is common to all. This greatly expands the scope for cross-country and cross-period mobility comparisons while preserving a single concept of relative mobility and immobility.

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Online Appendix

Appendix A. Discussion and Proofs for Section 2

A.1. Copula Uniqueness

Sklar's Theorem (1959) states that the copula associated with a joint distribution is unique if and only if marginal distributions are continuous. When the random variables take on discrete values or are drawn from distributions with mass points, the copula is generically not uniquely determined. Instead, it is only unique on the set of ranks the marginals actually take (Thm 2.3.3, Nelsen 2006). There are infinitely many valid extensions for values outside the observed support of ranks.

Thus, even if mass points are only at the boundary of support (as in the case of censored or truncated income observations), the copula is not unique. The results in this paper do not, unless stated otherwise, rely on continuity or differentiability of the copula. However, they do implicitly assume uniqueness of the copula. Therefore, discreteness represents a problem as it leads to indeterminacy.

Addressing this indeterminacy is referred to as an *extension* of the copula. Unfortunately, extensions are not necessarily guaranteed to preserve the concordance order. Thus, the choice of extension is not without loss. There are three solutions to this problem for a given empirical copula:

(i) Use a checkerboard extension (Genest and Nešlehová 2007) which is piecewise bilinear

(ii) Consider Carley (2002) bounds on copulas

(iii) Approximate the discrete outcomes with a specific parametric copula family

Solution (iii) is the easiest solution, but also requires the strongest assumptions making it typically unappealing. Solution (ii) would replace inequalities on the copula required for the concordance order with worst-case inequalities comparing upper and lower bounds. This is extremely flexible, but can be limiting when the bounds are not sharp. Finally, solution (i) restricts the set of admissible extensions and selects a bilinear interpolation on the density. This guarantees uniqueness and, as shown by Propositions 11 and 13 in Genest and Nešlehová (2007), the checkerboard extension is the unique extension that preserves the concordance order of the original joint-distributions. Consequently, I assume that discreteness is addressed through checkerboard extension of the copula. While this is a mild assumption, equivalent results can be obtained under either alternative assumption.

A.2. Proofs

Proof of Lemma 1. Let $u1 = v1 = 0$, $u_2 = u$ and $v_2 = v$. Then the boundary conditions of the copula give

$$C(0, v) = C(u, 0) = C(0, 0) = 0 \tag{A.1}$$

for all copulas. Thus Assumption 1 is equivalent to

$$\Delta C(u, v) \geq 0, \tag{A.2}$$

which is identical to Definition 1. □

Appendix B. Proofs for Section 3

Proof of Proposition 1. The population value of β is given by $cov(\ln y_i^k, \ln y_i^p)/var(\ln y_i^p)$. The denominator is constant and the numerator is increasing relative to the concordance ordering as a direct consequence of Lemma 3 in Lehmann (1966) (see also Lemma

2, Hoeffding 1940). Similarly, the population value of ρ is increasing relative to the concordance ordering from Corollary 3.2. in Tchen (1980). □

Proof of Proposition 2. In the parametric case, the conditional expectation is given by (substituting the closed form value of α):

$$CER = 0.5 + \rho(R - 0.5) \tag{B.1}$$

which is increasing in ρ above the median ($R > 0.5$) and decreasing below ($R < 0.5$). Consequently, given ρ is increasing relative to the concordance ordering (Proposition 1), the stated inequalities follow. This does not rely on Assumption 1.

Turning to the non-parametric case, the CER is monotonically increasing if and only if $C_{22} \leq 0$. Define

$$D(v) = \int_0^1 u\, \partial_v C^A(u, v)\, du, - \int_0^1 u\, \partial_v C^B(u, v)\, du, (v).$$

Then there exists a unique $v^* \in (0, 1)$ with $D(v^*) = 0$; equivalently, $\mathbb{E}^A[\, U \mid V = v\,]$ and $\mathbb{E}^B[\, U \mid V = v\,]$ cross exactly once. Assume the following regularity assumption: each section $v \mapsto C_i(u, v)$ is concave.[30] Therefore $D(v)$ is concave on $[0, 1]$.

First, consider the left tail of v. We establish that $D(v) > 0$ in the neighborhood of $v = 0$. Fix $\varepsilon \in (0, 1)$. Using $C^A(u, 0) = C^B(u, 0) = 0$ and $C^A(u, \varepsilon) \geq C^B(u, \varepsilon)$ for every u gives

$$\int_0^{\varepsilon} D(v)\, dv = \int_0^1 u\big[C_1(u, \varepsilon) - C_2(u, \varepsilon)\big]\, du > 0 \quad (\varepsilon > 0).$$

$D(0) = 0$ and D is continuous, thus some $v_0 \in (0, \varepsilon)$ satisfies $D(v_0) > 0$. Equivalently, in

[30]It is often possible to verify this concavity via monotone-likelihood-ratio or TP_2 properties in models that admit densities or for many parametric copula family. This guarantees uniqueness of the root.

the neighborhood of $v = 1$ $u - C^A(u, 1-\varepsilon) \leq u - C^B(u, 1-\varepsilon)$, hence

$$\int_{1-\varepsilon}^{1} D(v)\, dv < 0.$$

Thus, there exists $v_1 \in (1-\varepsilon, 1)$ with $D(v_1) < 0$. D is continuous, positive at v_0, and negative at v_1; by the Intermediate Value Theorem it has a root $v^* \in (v_0, v_1)$. Under the regularity assumption, D is concave and a zero can occur *at most once*. Hence v^* is the unique crossing. □

Proof of Proposition 3. The transition probabilities can be rewritten using the conditional CDF. For the positive diagonal case we have

$$TP\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right] = \frac{Pr(R^K \leq \tau^k \cap R^P \leq \tau^p)}{Pr(R^P \leq \tau^p)} = \frac{C(\tau^k, \tau^p)}{\tau^p}, \tag{B.2}$$

and

$$TP\left[R^K > \tau^k \mid R^P > \tau^p\right] = \frac{Pr(R^K > \tau^k \cap R^P > \tau^p)}{Pr(R^P > \tau^p)} = \frac{1 - \tau^p - \tau^k + C(\tau^k, \tau^p)}{1 - \tau^p}, \tag{B.3}$$

with the inequality in the proposition satisfied iff

$$C^A(\tau^k, \tau^p) \geq C^B(\tau^k, \tau^p)\ \forall \tau^k, \tau^p \in [0, 1]\,, \tag{B.4}$$

which is identical to the definition of increased concordance.

For the off-diagonal case we have

$$TP\left[R^K > \tau^k \mid R^P \leq \tau^p\right] = 1 - TP\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right], \tag{B.5}$$

and

$$TP\left[R^K \leq \tau^k \mid R^P > \tau^p\right] = 1 - TP\left[R^K \leq \tau^k \mid R^P > \tau^p\right], \tag{B.6}$$

with the stated inequality following directly from the results established in the positive diagonal case.

Turning to directional rank mobility. Fix $\tau \in (0,1]$ and an s such that $1-\tau \geq s \geq 0$. First, note that we can express the following conditional density through Bayes' rule

$$Pr(R^K - R^P > s \mid R^P \leq \tau) = \frac{Pr(R^K - R^P > s \cap R^P \leq \tau)}{Pr(R^P \leq \tau)} = \frac{Pr(R^K > R^P + s \cap R^P \leq \tau)}{Pr(R^P \leq \tau)}, \quad \text{(B.7)}$$

where the numerator is the probability that both conditions hold. This can be rewritten as

$$Pr(R^K > R^P + s \cap R^P \leq \tau) = \int_0^\tau Pr(R^K > t + s | R^P = t)\, dt, \quad \text{(B.8)}$$

which uses the fact that ranks are uniformly distributed and bounded between 0 and 1. The conditional density is well-defined (Theorem 2.2.7., Nelsen 2006).

As $Pr(R^P \leq \tau) = \tau$ and is constant across copulas, the stated inequality is identical to establishing the inequality pointwise on the integrand in equation (B.8).

Let

$$\begin{aligned} g_h(x) \equiv Pr(R^K > x + s \mid x < R^P \leq x + h) &= 1 - Pr(R^K \leq x + s \mid x < R^P \leq x + h) \\ &= 1 - \frac{Pr(R^K \leq x + s\, , x < R^P \leq x + h)}{h}, \end{aligned}$$

where the alternative definition is a direct application of Bayes' rule. The integrand in equation (B.8) can then be expressed as

$$Pr(R^K > t + s | R^P = t) = \lim_{h \downarrow 0} g_h(t), \quad \text{(B.9)}$$

where the function $g_h(\cdot)$ is continuous in h by uniform continuity of the copula (corollary 2.2.6., Nelsen 2006). Note that $C(u, v+h) - C(u, v) = Pr(R^K \leq u\, , v < R^P \leq v + h)$. We can then express $g_h(x)$ in terms of copulas as

$$g_h(x) = 1 - \frac{C(x+s, x+h) - C(x+s, x)}{h} = 1 - \frac{Pr(R^K \leq x+s\,, X < R^P \leq x+h)}{h}. \tag{B.10}$$

Assumption 1 guarantees $\Delta C(u, v+h) \geq \Delta C(u, v)$ and

$$C^A(u, v+h) - C^A(u, v) \geq C^B(u, v+h) - C^B(u, v) \tag{B.11}$$

$$\longrightarrow \frac{C^A(u, v+h) - C^A(u, v)}{h} \geq \frac{C^B(u, v+h) - C^B(u, v)}{h}. \tag{B.12}$$

Substituting (B.12) into (B.10), establishes

$$g^B(x) \leq g^A(x) \longrightarrow \lim_{h \downarrow 0} g^B(x) \leq \lim_{h \downarrow 0} g^A(x) \longrightarrow \int_0^\tau \lim_{h \downarrow 0} g^B(t)\, dt \leq \int_0^\tau \lim_{h \downarrow 0} g^A(t)\, dt, \tag{B.13}$$

thus $Pr^B(R^K > R^P + s \cap R^P \leq \tau) \leq Pr^A(R^K > R^P + s \cap R^P \leq \tau)$. □

Proof of Proposition 4. Fields and Ok Measure. The aggregate income movement per capita is equivalent to

$$\int\int \psi\left(Y^K, Y^P\right) f\left(Y^K, Y^P\right) dY^K dY^P \quad \text{where } \psi(x, y) = |x - y|, \tag{B.14}$$

where the function $\psi(x, y)$ is a univariate convex function. The result then follows directly as an application of Theorem 9.A.18 in Shaked and Shanthikumar (2007) or Corollary 2.3 (and example 2) in Tchen (1980). Expected absolute deviations across generations are decreasing in the concordance order. The proof in the log-case (Fields and Ok 1999b) is identical. The Fields and Ok (1996, 1999b) measures can be decomposed into structural mobility (an aggregate growth term),

$$\int |Y^K| f^k(Y^K) dY^K - |Y^P| f^p\left(Y^P\right) dY^P, \tag{B.15}$$

which is identical under C^A and C^B with constant marginals as well as a transfer or exchange term

$$2 \times \int \int_{(Y^K, Y^P) \in S} |Y^K - Y^P| \, f(Y^K, Y^P) \, dY^K \, dY^P \,, \tag{B.16}$$

where the set S selects individuals who are winners or losers. These are dynasties that experience reversals relative to the direction of aggregate income change:

$$S = \begin{cases} \{(Y^K, Y^P) : Y^K < Y^P\}, & \text{if} \quad \overline{Y}^K > \overline{Y}^P \\ \{(Y^K, Y^P) : Y^K > Y^P\}, & \text{if} \quad \overline{Y}^K < \overline{Y}^P \end{cases} . \tag{B.17}$$

It follows that this measure of exchange mobility is decreasing in the supermodular order and thus the concordance order with common marginals.

Cowell and Flachaire Measure The aggregate mobility measure is

$$\int \int \psi_\alpha \left(Y^K, Y^P\right) f \left(Y^K, Y^P\right) d\, Y^K d\, Y^P \,, \tag{B.18}$$

where the function $\psi_\alpha (x, y) = {}^1\!/_{\alpha(\alpha-1)} \, ({}^x\!/_{\overline{x}})^\alpha \, ({}^y\!/_{\overline{y}})^{1-\alpha}$ has the following cross-partial derivative:

$$\frac{\psi_\alpha (x, y)}{\partial x \partial y} = -\frac{1}{\overline{x}^\alpha \overline{y}^{1-\alpha}} x^{\alpha-1} y^{-\alpha} \leq 0 \,. \tag{B.19}$$

Thus the function is submodular. The result then follows directly as concordance is equivalent to the supermodular order when marginals are constant (Tchen 1980).

Shorrocks Measure Note that the elements stochastic matrix Ω are given by

$$\Omega_{i,j} = Pr \left(\frac{j-1}{q} < R^K \leq \frac{j}{q} \mid \frac{i-1}{q} < R^P \leq \frac{i}{q} \right) \tag{B.20}$$

$$= \frac{1}{q} \left(C\left(\frac{i}{q}, \frac{j}{q}\right) - C\left(\frac{i-1}{q}, \frac{j}{q}\right) - C\left(\frac{i}{q}, \frac{j-1}{q}\right) + C\left(\frac{i-1}{q}, \frac{j-1}{q}\right) \right), \tag{B.21}$$

and the trace is

$$trace\,(\mathcal{Q}) = \sum_{i=1}^{q} \mathcal{Q}_{i,i}. \tag{B.22}$$

It follows that differences in the Shorrocks measure satisfy

$$\begin{aligned}
M^B - M^A \propto & \, trace\left(\mathcal{Q}^A\right) - trace\left(\mathcal{Q}^B\right) \\
= & \frac{1}{q}\sum_{i=1}^{q}\left(C^A\left(\tfrac{i}{q},\tfrac{i}{q}\right) - C^A\left(\tfrac{i-1}{q},\tfrac{i}{q}\right) - C^A\left(\tfrac{i}{q},\tfrac{i-1}{q}\right) + C^A\left(\tfrac{i-1}{q},\tfrac{i-1}{q}\right)\right) \\
& - \frac{1}{q}\sum_{i=1}^{q}\left(C^B\left(\tfrac{i}{q},\tfrac{i}{q}\right) - C^B\left(\tfrac{i-1}{q},\tfrac{i}{q}\right) - C^B\left(\tfrac{i}{q},\tfrac{i-1}{q}\right) + C^B\left(\tfrac{i-1}{q},\tfrac{i-1}{q}\right)\right) \\
= & \frac{1}{q}\sum_{i=1}^{q}\left(\Delta C\left(\tfrac{i}{q},\tfrac{i}{q}\right) - \Delta C\left(\tfrac{i-1}{q},\tfrac{i}{q}\right) - \Delta C\left(\tfrac{i}{q},\tfrac{i-1}{q}\right) + \Delta C\left(\tfrac{i-1}{q},\tfrac{i-1}{q}\right)\right) \geq 0,
\end{aligned}$$

where the final inequality follows directly from Assumption 1. □

Proof of Proposition 5. This is immediate as an application of the closure properties of the concordance order. See Theorem 9.A.1 Shaked and Shanthikumar (2007). □

Proof of Corollary 1. This is immediate as an application of the closure properties of the concordance order. See Theorem 9.A.1 Shaked and Shanthikumar (2007). □

Appendix C. Proofs for Section 6

Proof of Lemma 2. As $I^{*A}(Y^P) \geq I^{*B}(Y^P)$ for all Y^P (Assumption C) and f is increasing in I, we have

$$H^{K,A} \mid Y^P = y \;\; \succeq_{\text{FOSD}} \;\; H^{K,B} \mid Y^P = y \qquad \forall y.$$

Applying each economy's own CDF, the rank variables satisfy

$$F_{R^{K,A}|Y^P}(r \mid y) \;\leq\; F_{R^{K,B}|Y^P}(r \mid y) \qquad \forall r \in (0,1),\; \forall y, \tag{C.1}$$

as the same H^K is associated with a lower-rank and a lower probability

Integrating inequality (C.1) gives the bivariate joint density as Y^P has the same marginal density f_{Y^P} in both economies. Hence

$$(R^K, Y^P)^A \succeq (R^K, Y^P)^B.$$

Which is equivalent to the concordance ordering as the copula is preserved under monotone transformations.

□

Proof of Lemma 2. For any Y^P, let $V(I;\lambda) = U\left(C^P\right) + \delta\mathbb{E}_\theta\left[U\left(W_\lambda(H^K)\right)\right]$ denote the value of the objective function for choice I and parameter λ controls the counterfactual. The proof follows from Topkis theorem under increasing differences. If the cross-partial (or discrete analogue) satisfies

$$\frac{\partial^2 V}{\partial I \partial \lambda} = \frac{\partial}{\partial \lambda}\left[\delta \frac{\partial f(I, H^P)}{\partial I}\mathbb{E}_\theta\left[U'\left(W_\lambda(H^K)\right) W'_\lambda(H^K)\right]\right] \geq 0, \tag{C.2}$$

then investment shifts outwards as the policy parameter λ increases. Under mild regularity conditions on general equilibrium effects,[31] it is easy to verify that the stated counterfactuals satisfy increasing differences as the production function is supermodular and the wage schedule is strictly increasing. Thus $I^*(Y^P;\lambda') \leq I^*(Y^P;\lambda) \quad$ whenever $\lambda' < \lambda$.

□

[31] If the wage schedule does not change there is no contribution to the partial derivative. The technical condition above is the minimum requirement, but stronger conditions such as single crossing of wage schedules across regimes or a simple outward expansion imply the condition holds. In quantative models this typically holds.

Appendix D. Construction of the Replication Dataset

The construction of the data follows Jácome, Kuziemko, and Naidu (2025). I briefly summarize the key features of their approach here and refer the interested reader to Jácome, Kuziemko, and Naidu (2025).

Data sources. I pool the fifteen U.S. surveys identified by Jácome, Kuziemko, and Naidu (2025) that report (i) respondents' current family income, (ii) father's occupation when the respondent was growing up, (iii) respondent race, and (iv) birthplace or childhood region (South vs. non-South). The surveys include American National Election Studies, General Social Survey, the Panel Study of Income Dynamics (specifically 1997 and 2017 waves), Occupational Changes in a Generation 1962 and 1973 surveys, the National Longitudinal Surveys, and others listed in Appendix E of their paper.

I use IPUMS census extracts from 1910 to 2019 (American Community Survey). These extracts differ from the original analysis which uses the full 1940 census available on the NBER server. They may also differ in other years due to differences in the size of the census-extracts selected.

Sample. I retain U.S-born men and women aged 30–50, the window that best approximates permanent income while minimising life-cycle bias . Respondents must have non-missing family income, race, region, and father's occupation. Foreign-born individuals are excluded because parental incomes are imputed from U.S. sources.

Respondent income. Each survey's family-income question is harmonised into 10–12 real-1950-dollar bins; continuous responses are recoded to the bin mid-point. Bottom-coded observations (including 0) are set to 0.75 × the upper boundary of this lowest bin and those in the top-bin are set to 1.25 × lower-bin boundary.

Parental income imputation. Fathers' (and, where available, mothers') occupations are mapped into 28 broad categories (e.g., skilled crafts, farm operators) . Predicted parental family income is the mean household income in matching occupation × race × South cells drawn from historical micro-data: 1901 Cost of Living Survey & 1900 Census (early cohorts), full-count 1940 Census plus the 1936 Expenditure Survey, and 1960–1990 Censuses . For farmers and self-employed, incomes are adjusted following Collins and Wanamaker (2022); non-working fathers receive values imputed from contemporaneous Census means. Father race is proxied by respondent race; father region by respondent childhood region.

Weighting. Where surveys supply sampling weights, I re-centre them to mean 1. I then adjust all surveys so that each birth-decade cell matches Census race-sex population shares (white men/women, Black men/women); surveys lacking weights receive weight equal to 1 before adjustment.

Cohorts. Decade birth cohorts 1910s–1970s are used. For each respondent (a) log family income and (b) percentile rank in the pooled income distribution are computed; parental income is treated analogously, yielding comparable marginals across cohorts.

These steps reproduce the long-run, nationally representative parent–child dataset used in the original study and permit the replication of Figure 1 in their paper (Panel (A), Figure 8 here).