Sufficient Statistics for Economic Mobility: When Do Measures Agree?*

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April 18, 2026

Abstract

Empirical studies of intergenerational mobility use different statistics—elasticities, rank–rank slopes, transition matrices, axiomatic indices—that sometimes produce conflicting conclusions. I show that a single property determines when they agree: concordance, capturing how rank-preserving the parent–child joint distribution is. When one economy is everywhere more rank-preserving, all standard measures agree on which is more mobile. Any change in a mobility measure decomposes into a dependence component and an inequality component; rank-based measures isolate dependence while level-based measures conflate both. Concordance arises naturally in a Becker–Tomes–Loury model and is robust to proxies and measurement error. Twentieth-century U.S. data confirm the predictions, rank-based measures co-move while the intergenerational elasticity diverges as inequality rises.

Keywords: Concordance, Mobility, Human Capital
JEL classification: D31, I32, J62
Acknowledgments: This project was supported by SSHRC through Insight Development Grant Number 430-2022-00394. All errors and omissions remain the responsibility of the author. I am grateful to Roy Allen, Richard Audoly, Yonatan Berman, Raj Chetty, Jim Davies, Jamie Fogel, Matthew Gudgeon, Nathaniel Hendren, Lawrence Katz, Lance Lochner, Sergio Ocampo, Martin O'Connell, Cormac O'Dea, Matthew Staiger, and Jan Stuhler for their comments and suggestions along with seminar participants at Opportunity Insights.

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1. Introduction

Researchers measure economic mobility in a variety of ways—earnings elasticities, rank-rank correlations, transition matrices or axiomatic indices—to document differences across countries (e.g., Corak 2013; Jäntti et al. 2006), regions (e.g., Chetty et al. 2014a; Corak 2020), and over historical periods (e.g., Feigenbaum 2018; Collins and Wanamaker 2022).[1] In many contexts, these measures, or summary statistics, produce consistent rankings of mobility (e.g., Katz and Krueger 2017, Berman 2022, Deutscher and Mazumder 2023), but in others the choice of measure changes the conclusion.

Even among purely rank-based measures, agreement is not guaranteed. In Chetty et al. (2014a), geographic differences in mobility across U.S. commuting zones are imperfectly correlated across measures, producing conflicting rankings. Whether Salt Lake City is more mobile than New York, for instance, depends on on which feature of the transition matrix is used to measure mobility.[2] The same pattern appears in long-run U.S. trends, where the intergenerational elasticity and rank–rank slope diverge as inequality rises (Jácome, Kuziemko, and Naidu 2025), and in cross-country comparisons (Blanden 2013)—settings where changing marginal distributions introduce an additional source of disagreement. This raises two fundamental questions: when should we expect standard mobility measures to deliver the same ranking of economies, and when should disagreement be interpreted as substantive or indicative of underlying economic mechanisms?

This paper proposes a framework to organize these agreements and disagreements. When the parent-child joint distribution in one economy is everywhere more rank-preserving—i.e., when parent and child ranks become more closely aligned throughout the distribution—a researcher reporting intergenerational elasticities will reach the same conclusion about relative mobility as one reporting rank-rank slopes, transition

[1]Researchers use mobility measures to study the dependence of income across and within generations (e.g., Solon 1992; Chetty et al. 2014b), as well as wealth (e.g., Boserup, Kopczuk, and Kreiner 2018; Fagereng et al. 2020; Fagereng, Mogstad, and Rønning 2021; Audoly et al. 2024), consumption (e.g., Jappelli and Pistaferri 2006), health (e.g., Halliday 2023; Black et al. 2024), education (e.g., Black and Devereux 2011), socio-emotional skills (Attanasio, De Paula, and Toppeta 2025), or occupational prestige (Mazumder and Acosta 2015; Olivetti and Paserman 2015; Song et al. 2020; Haeck and Laliberté 2025). They are used in positive analysis, normative analysis (e.g., Shorrocks 1978; Atkinson and Bourguignon 1982), evaluating the impacts of social policy (e.g., Chetty and Hendren 2018a,b), and as validation or targets for structural models of economic behaviour (e.g., Abbott et al. 2019; Bolt et al. 2025; Daruich 2018).

[2]Their Table III reports transition probabilities defined over national income quintiles, so cross-CZ differences partly reflect where each CZ sits in the national distribution rather than within-CZ dependence alone. In Appendix A, I reconstruct within-CZ copulas from the published transition matrices and show that measure disagreement persists after isolating the dependence structure.

probabilities, or an axiomatic index (e.g., Shorrocks 1978). Measures are then *sufficient statistics* for overall mobility and become substitutes. This observation explains why studies that report multiple measures, either as distinct objects of interest or robustness, typically find broadly consistent results.

This substitutability has immediate practical implications. Researchers have advanced different arguments for preferring elasticities, rank–rank slopes, or transition matrices on grounds of interpretability, robustness, or transparency.[3] When economies can be ordered by the rank-strengthening alignment above, these debates are moot as all measures agree on which economy is more mobile, and researchers can select based on data availability or inference concerns without fear of missing substantive information.

Importantly, this rank-strengthening property is empirically plausible and is not an arbitrary statistical restriction—applied mobility research already rules out precisely the kinds of rank-reversing patterns that violate the order I study. Bounding exercises for absolute mobility impose the "*intuitive requirement that children from higher-income families are less likely to have lower incomes*" (Chetty et al. 2017 pg. 401; see also Berman 2022; Manduca et al. 2024), and work using transition matrices routinely treats substantial anti-diagonal mass—where high-income parents systematically produce low-income children and vice versa—as empirically implausible (Shorrocks 1978; Jäntti and Jenkins 2015). I formalise these shared restrictions as a uniform, rank-improving shift of the parent–child copula.

Substitutability follows from this single testable dependence order on the parent–child copula, known as concordance (Tchen 1980). The copula captures the dependence structure between parent and child ranks, independent of the marginal income distributions in each generation. The concordance order is empirically testable using standard moment-inequality tests, theoretically grounded in standard models of human capital investment, and satisfied by many parametric joint distributions commonly used in applied work. Moreover, the concordance order's implications are robust to measurement error and the use of proxies (such as education or occupation) in place of lifetime income, strengthening the practical value of these sufficiency

[3] The intergenerational elasticity is widely used because it is interpretable, model-consistent, and comparable to an established evidence base (Black and Devereux 2011), though it is sensitive to life-cycle bias (Haider and Solon 2006; Nybom and Stuhler 2017). Rank–rank slopes are robust and unit-free (Chetty et al. 2014a). Transition matrices are transparent (Jäntti and Jenkins 2015) with minimal data requirements, but depend on discretization (Cowell and Flachaire 2018).

results.[4] The role of concordance in mobility measurement parallels that of Lorenz dominance in inequality measurement: just as Pigou-Dalton transfers generate an ordering under which all transfer-sensitive inequality measures agree (Atkinson 1970), concordance-increasing exchanges generate an ordering under which all pure exchange-mobility measures agree.

This framework also reveals why measures diverge. Any change in a mobility measure decomposes into a component due to changing dependence (the copula) and a component due to changing inequality (the marginal distributions). Rank-based measures—rank–rank slopes, transition matrices, the Shorrocks index—have a zero marginal component by construction, since ranks are uniformly distributed regardless of the underlying income distribution. Concordance therefore orders these measures even when comparing economies with very different levels of inequality. Level-based measures such as the intergenerational elasticity conflate both sources of variation: even when the dependence component is ordered by concordance, rising inequality can offset or reverse the overall trend. This immediately explains why measures diverge in Jácome, Kuziemko, and Naidu (2025) as post-1950 U.S. rank-dependence continued weakening, but rising inequality dominates the intergenerational elasticity, masking the continued decline in persistence.

Concordance-increasing shifts arise naturally in Becker and Tomes (1979)-Loury (1981) models under standard economic forces (Section 8). When economic primitives raise the return to human capital or improve skill-formation technologies optimal investment shifts outward at every income level. This induces exactly the copula changes described above and all mobility measures decline in tandem. This provides a clear economic interpretation, the "*measures-as-substitutes*" result applies whenever shocks uniformly strengthen the parent-child income link. This is regardless of whether shocks occur through educational expansion, technological change, or policy reforms.

Not all economically relevant changes take this form, however. Policies or shocks that improve outcomes for some parts of the distribution while weakening the parent–child link elsewhere—such as targeted interventions for disadvantaged families that coincide with increased sorting among elites—can cause the concordance order to fail. More generally, not all copulas can be ordered by concordance. In such cases,

[4] When concordance orders economies, measures are ordinal substitutes: one ranking reveals all others. Measures remain distinct objects (the intergenerational elasticity and rank-rank slope take different numerical values and units), but concordance guarantees they agree on which economy is more mobile—the primary question in most comparative applications—even as cardinal magnitudes differ. Thus, the term "Sufficient Statistic" is not used in the Fisher-information sense of classical statistics.

different measures disagree because they weight different regions of the joint distribution. I make this transparent by showing precisely which regions each measure emphasises and demonstrate that disagreement is not an artefact of measure choice but generic. When concordance fails, there always exist standard measures that produce conflicting rankings. For these cases, I propose a structured reporting protocol: report local rank–rank slopes in the regions where concordance holds and fails, supplemented by transition probabilities at standard thresholds, so that readers with different preferred measures can assess the implications for their own conclusions. When a single ranking is nevertheless required, I discuss principled completions of the partial order.

Section 7 applies the framework to twentieth-century U.S. mobility trends, an application that incorporates both rank-based measures and changes in inequality. The theory's predictions are borne out in the data. Empirically, these rank-based measures (rank–rank slopes, transition probabilities, directional mobility) move together in a wave-like pattern. This coherence holds despite institutional changes, varying data quality, and the use of imputed parental incomes, validating concordance as an empirically relevant organising principle. Testing concordance shows it holds for all long-run cohort comparisons but is formally rejected for two adjacent pairs. These are precisely the pairs where rank-based measures disagree. In contrast, level-based measures (the IGE) diverge as inequality rises, exactly as the framework predicts when marginal distributions change. Moreover, I show that the effect of inequality dominates for level-based measures, highlighting a fundamental source of disagreement.

This paper is organised as follows. Sections 2 and 3 introduce the concordance framework and establish that all standard exchange mobility measures are monotone in concordance. Section 4 extends results to settings with changing marginal distributions. Sections 5 and 6 examine when concordance fails and provide practical guidance, including the reporting protocol and completions of the partial order. Section 7 applies the framework to twentieth-century U.S. data. Section 8 microfounds concordance in a model of endogenous human capital formation. Section 9 concludes.

1.1. Related Literature

A large body of work proposes and estimates mobility measures (surveys include Black and Devereux 2011, Deutscher and Mazumder 2023). These studies ask which measure best captures underlying mobility. This paper asks the reverse: under what conditions do these measures deliver the same ordering of economies?

The closest theoretical work is D'Agostino and Dardanoni (2009), who axiomatise concordance as a measure of rank mobility in a discrete transition-matrix setting and cardinalise by scaling against a maximal benchmark. Atkinson and Bourguignon (1982) study how dependence shapes utilitarian welfare rankings in multidimensional inequality, using restrictions on the joint distribution of economic status that are formally similar to those employed here. This paper differs in three respects. First, instead of proposing a new index, I show that concordance serves as a common dependence condition linking many mobility measures already used in applied work—regression-based, transition-matrix, and axiomatic. Second, I extend the results beyond discrete matrices to continuous joint distributions and to environments with proxies and classical measurement error. Third, I provide a microeconomic interpretation in a Becker–Tomes–Loury model, making explicit which economic forces generate concordance-increasing shifts. Whereas D'Agostino and Dardanoni axiomatise a particular mobility index and Atkinson and Bourguignon study welfare comparisons in multidimensional settings, I study the positive properties of existing empirical estimands and the conditions under which they can be treated as substitutes.

I focus on the properties of *estimands* rather than their estimators. A related literature considers inference and measurement error for rank-rank specifications (Chetverikov and Wilhelm 2023; Kitagawa, Nybom, and Stuhler 2018), identification under general family structures (Collado, Ortuño-Ortín, and Stuhler 2023; Espín-Sánchez, Ferrie, and Vickers 2023), and biases in standard estimators (Haider and Solon 2006; Nybom and Stuhler 2017). My contribution is complementary, I characterise when population parameters from different measures deliver identical orderings, regardless of how those parameters are estimated, and show in Section 3 that concordance orderings survive measurement error so that sufficiency results apply even when researchers observe noisy proxies.

This paper studies relative (exchange) mobility and is silent on absolute mobility measures such as the fraction of children exceeding the median income of the parent generation (Katz and Krueger 2017). Ray and Genicot (2023) show that panel-independent mobility measures can be constructed when welfare depends only on growth progressivity. The concordance framework complements this work: while Ray and Genicot characterise planner preferences over upward mobility, I characterise the dependence structure determining relative positional mobility. Some absolute mobility bounds (Chetty et al. 2017; Berman 2022; Manduca et al. 2024) impose copula restrictions; concordance clarifies when those restrictions are empirically plausible.[5]

This paper relates to work studying sorting through concordance properties. Gola (2021), Anderson and Smith (2024), and Boerma et al. (2023) derive concordance orderings as equilibrium outcomes of assignment problems and use them to measure sorting in the labour market. Chiappori et al. (2025) axiomatise an odds-ratio index for assortative mating that is both necessary and sufficient for concordance in discrete contingency tables; their normalised-trace index is a direct transformation of the Shorrocks (1978) mobility measure. I apply similar tools to a different question, but the implication that concordance makes measure choice redundant carries over directly to measures of assortative mating and matching in marriage and labour markets. Section 8 shows concordance arises endogenously in a Becker–Tomes–Loury investment model, paralleling the endogenous concordance derived in matching models, and links to work studying causal impacts on intergenerational mobility (Chetty et al. 2026; Nakamura, Sigurdsson, and Steinsson 2022).

2. Framework: Copulas and Concordance

This paper considers the following setting. A researcher is comparing economic mobility between economies. Specifically, the focus is on the joint behavior of parent–child (or period-to-period) outcomes and how that dependence shapes measures of mobility. I begin by defining the relevant joint distributions and providing the definition of the concordance order.

Let (Y_K, Y_P) denote child and parent outcomes with joint CDF $F(Y_K, Y_P)$ and marginals $F_K(Y_K)$ and $F_P(Y_P)$. The key insight is that relative mobility—how children's positions relate to parents' positions—depends entirely on the dependence structure between these outcomes, not their levels or dispersion.

This dependence structure is captured by the copula. Ranks, $R_K = F_K(Y_K)$ and $R_P = F_P(Y_P)$, are uniformly distributed on $[0, 1]$ and map from incomes to the unit interval. By Sklar's Theorem (1959), the joint distribution can be decomposed as

$$F(Y^K, Y^P) = C\left(F_K(Y^K), F_P(Y^P)\right), \tag{1}$$

[5] I focus on bivariate parent-child distributions. Orthant orders (Shaked and Shanthikumar 2007) generalise the concordance order to multivariate settings and Audoly et al. (2024) adopt hierarchical clustering to summarise high-dimensional wealth and income histories. The bivariate focus isolates the core dependence structure in most empirical work and simplifies exposition.

where $C : [0,1]^2 \to [0,1]$ is the copula or joint distribution of ranks. The *copula alone* encodes all rank-dependence between generations, independently of marginal distributions.[6] This separation means mobility measures operating on relative positions are simply alternative summaries of the same underlying copula.[7]

To isolate dependence, the main analysis will fix marginal distributions and focus solely on changes in the copula. For instance, imagine a thought experiment in which the U.S. marginal income distributions remain constant, but the parent–child rank-dependence structure varies. Under this restriction, each mobility measure becomes an alternative summary of the same copula—that is, of how often "high" outcomes for one generation coincide with "high" outcomes for the next. Section 4 shows how relaxing the constant-marginals assumption affects the results.

2.1. The Concordance Order

I begin by illustrating changes in dependence with an example which assumes outcomes take on discrete values for simplicity, then formalize the concordance order.

Figure 1 presents a grid of parent–child quintile transition matrices. Moving rightward or downward increases concordance: probability mass shifts from anti-diagonal (rank-reversing) to diagonal (rank-preserving) cells. For instance, the top-left panel shows perfect discordance, the center shows independence, and the bottom-right shows perfect concordance.

The statistical property of concordance formalizes the intuitive idea that "large" values of one random variable tend to coincide with "large" values of another, a form of stochastic dominance for dependence in joint-distributions (e.g., Kirkegaard 2017). In the context of intergenerational mobility copulas, concordance is higher if parental ranks are more likely to be realised with higher child ranks. Importantly, it appeals to an implicit sense of mobility. For example, the bounding exercise in Chetty et al. (2017) assumes the copula satisfies the "*intuitive requirement that children from higher-income families are less likely to have lower incomes*" (pg. 401). Concordance formalises these types of intuitions in a mathematically precise and testable stochastic ordering over

[6] I treat the copula and marginals as observed, abstracting from inference (see Chetverikov and Wilhelm 2023). Appendix B discusses uniqueness of the copula for non-continuous cases.

[7] This property is exploited by Chetty et al. (2017) and Manduca et al. (2024) who impose copula restrictions to bound absolute mobility when panel data are unavailable. I exploit the same logic: summarizing relative mobility via the copula alone. Copulas also accommodate mass points (e.g., top-coded incomes or non-participation).

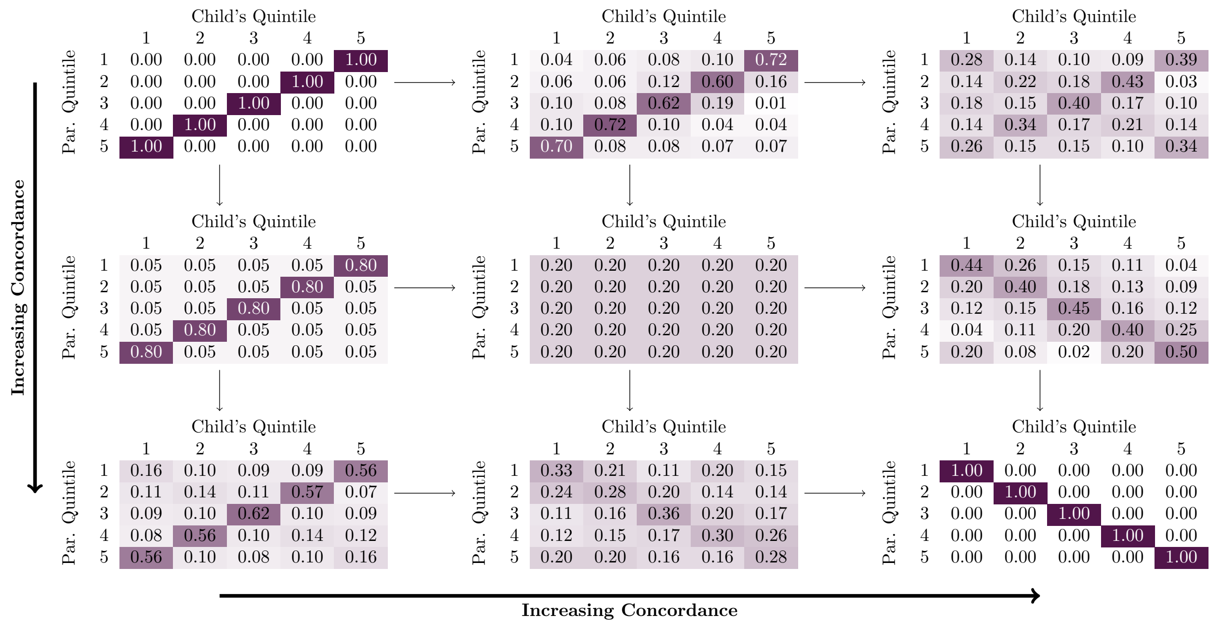


FIGURE 1. Concordance Ordering Among Transition Matrices

Notes: Moving rightward or downward increases concordance—probability mass shifts from off-diagonal (high-parent/low-child or vice versa) to diagonal (similar ranks across generations). Top-left: perfect rank-reversal (children systematically occupy opposite quintiles from parents). Center: independence (no association). Bottom-right: perfect rank-preservation (children inherit parental quintiles exactly). Empirical mobility estimates typically fall between the center and bottom-right panels.

copulas:[8]

DEFINITION 1 (Concordance Ordering (Yanagimoto and Okamoto 1969; Tchen 1980)). *Let C^A denote the copula, or joint distribution of ranks, in economy A and C^B denote the copula in economy B. The dependence between outcomes Y^K, and Y^P is said to be larger in concordance in economy A than economy B if and only if $C^A(u,v) \geq C^B(u,v)\ \forall\, u, v \in [0,1]$.*

This is denoted $C^A \succeq C^B$ or $A \succeq B$.

Note that higher concordance corresponds to lower mobility, as parent-child ranks become more strongly aligned: $C^A \succeq C^B$ implies less mobility in A than B. Thus, 'increasing concordance' and 'decreasing mobility' are synonymous. Section 7 provides a formal test constructed using moment-inequality procedures (Andrews and Shi 2013).

Formally, the concordance order captures two distinct concepts. First, the degree of monotone dependence or the tendency of random variables to cluster around the

[8]Throughout I will use superscript A and B to denote primitives and measures in different economies A and B, respectively. This ordering is sometimes alternatively known as the PQD ordering or point-wise copula ordering.

graph of any (measurable) monotone function $Y^K = f(Y^P)$ or $Y^P = g(Y^K)$. Second, the direction of monotonicity—whether the functions are monotone increasing or decreasing. The concordance order runs from maximum mobility (perfect rank-reversal between generations, the Fréchet lower bound)[9] to minimum mobility (perfect rank-preservation, the Fréchet upper bound).

Geometric interpretation. Any concordance increase can be decomposed into elementary 'switches': pick any rectangle in rank-space and shift probability from corners where parents and children have opposite relative positions (one high, one low) to corners where both are high or both are low[10] (Figure 1 illustrates these switches). The following elementary exchanges define a sequence of copula transformations ordering joint-densities by their concordance (see also Tchen 1980):

Definition 2 (Concordance Increasing Exchanges). *Let C^A be a copula. Pick $0 < u_1 < u_2 < 1$ and $0 < v_1 < v_2 < 1$, and let $\varepsilon > 0$ be small enough that the update below remains a valid copula. Define*

$$C^B(u, v) \;=\; C^A(u, v) + \varepsilon\Big(\mathbf{1}_{\{u\geq u_1,\, v\geq v_1\}} + \mathbf{1}_{\{u\geq u_2,\, v\geq v_2\}} - \mathbf{1}_{\{u\geq u_1,\, v\geq v_2\}} - \mathbf{1}_{\{u\geq u_2,\, v\geq v_1\}}\Big).$$

We say C^B is obtained from C^A by a concordance-increasing exchange *on the rectangle* $[u_1, u_2] \times [v_1, v_2]$.

This operation preserves the marginals, $C^B(u, 1) = u$ and $C^B(1, v) = v$, and moves probability mass from the two off-diagonal corners of the rectangle to the two diagonal corners. The reverse update with $\varepsilon < 0$ is concordance-decreasing and the operation can be confined to arbitrarily small rectangles ($u_2 - u_1 < \eta$ and $v_2 - v_1 < \eta$, for $\eta > 0$). Any concordance increase can be decomposed into a sequence of such exchanges. This provides a simple building block for increased concordance.

These exchanges are the mobility analogue of Pigou-Dalton transfers in inequality measurement. Just as Pigou-Dalton transfers generate Lorenz dominance, which is sufficient for all transfer-sensitive inequality measures to agree (Atkinson 1970), concordance-increasing exchanges generate the concordance order, which Section 3 shows is sufficient for all exchange-mobility measures to agree. The logic runs in three

[9]This is akin to maximal negative rank correlation and generalizes Prais (1955)'s origin independence which corresponds to the independence copula. This is analogous to Negative Assortative Matching as the reverse of Positive Assortative Matching rather than the absence of sorting.

[10]For finite economies with N dynasties, concordance has an equivalent discrete definition for dynasities: $A \succeq B$ if swapping any two parent-child pairs satisfies $(Y_i^K - Y_j^K)(Y_i^P - Y_j^P) > 0$.

steps: the elementary exchanges define a building block for increased dependence; iterating them produces the concordance partial order over copulas; and that partial order is sufficient to determine the rankings of all standard mobility measures simultaneously. Section 5 shows that when the concordance order fails—as Lorenz dominance can fail when income distributions cross—measures disagree because they weight different regions of the joint distribution, just as inequality indices disagree when Lorenz curves intersect.

Families of Distributions. A large class of multivariate distributions are ordered by their concordance. While the concordance order is nonparametric, many common parametric families impose concordance orderings as dependence increases (Joe 2014). Bivariate normal, elliptical, and Archimedean copulas (Clayton, Gumbel, Frank) all become more concordant as their dependence parameters increase. Similarly, multivariate extensions of the Singh and Maddala (1976) size-distribution for incomes, generalizing Champernowne (1952); Fisk (1961) and Pareto distributions have a parametric concordance order. The concordance order is stable under increasing marginal transformations and under discretization to finite transition matrices. Moreover, the order is preserved under convex combinations and weak limits: mixtures of more-concordant copulas remain more concordant, and pointwise limits of concordance-ordered sequences inherit the order (Shaked and Shanthikumar 2007).

3. Concordance Ordering and Sufficient Statistics For Exchange Mobility

Having established concordance as a key and intuitive dependence concept, I now turn to showing that this single ordering determines the rankings produced by all commonly used mobility measures—from regression-based to axiomatic indices, operating on ranks, logs, or income level. This demonstrates that concordance ensures substitutability. Thus, when it applies, researchers need not choose between competing metrics, as they all provide the same conclusion about relative mobility.[11] Building on Deutscher and Mazumder (2023)'s taxonomy and axiomatic contributions such as Shorrocks (1978), I focus on exchange mobility measures that depend solely on

[11]Thus, I use 'sufficient statistic' in the ordinal sense: concordance is sufficient to determine measure-rankings. This differs from the cardinal sense in classical statistics (Fisher information). When concordance orders economies, one measure reveals ordinal rankings of all others, but not their cardinal values. Thus a change in any measure is a sufficient statistic.

the joint-distribution. In applied work Y is most commonly income (e.g., Chetty et al. 2014b), although other measures such as wealth (e.g., Fagereng, Mogstad, and Rønning 2021; Audoly et al. 2024) or health (e.g., Halliday 2023) are also used. These coherence results are presented sequentially; formal proofs appear in Appendix C.

3.1. Regression

I begin by analysing two of the most widely used measures: the intergenerational earnings elasticity (IGE) and rank-rank regression slope. The IGE is the value of β, obtained from the following regression

$$\ln Y_i^K = \alpha + \beta \ln Y_i^P + \epsilon_i, \tag{2}$$

and the Rank-Rank regression slope, the value of ρ obtained from the following regression

$$R_i^K = \alpha + \rho R_i^P + \epsilon_i. \tag{3}$$

It is well known that the concordance order provides an ordering over covariances (Tchen 1980; Lehmann 1966) and spearman's rank correlation. Thus, with fixed marginals, implied mobility measures (e.g., $1 - \beta$) are unambiguously smaller in more concordant economies.

PROPOSITION 1 (Linear Regression Measures). *Economies A and B have identical marginals, but different rank dependence denoted by copulas C^A and C^B, respectively. If $C^A \succeq C^B$, then*

i. The intergenerational earnings elasticity (IGE) is ordered: $\beta^A \geq \beta^B$; and

ii. The Rank-Rank Slope is ordered: $\rho^A \geq \rho^B$.

Both increase relative to the concordance ordering.

Figure 2 illustrates Proposition 1, as concordance increases (blue diamonds to orange circles), realizations cluster more tightly around co-monotonicity in both log- and rank-space. A direct corollary is an order over correlation measures: the intergenerational correlation, $corr(\ln Y^K, \ln Y^P)$, and the rank correlation, $corr(R^K, R^P)$

also increase relative to the concordance ordering.[12]

In addition, Proposition 1 can be extended to consider mobility for subsets of the population. For example, children born to parents in a specific segment of the income distribution. In these cases, the assumption of a concordance order can be relaxed to apply 'locally' in a segment of the income distribution. In this case concordance need only hold on the relevant subdomain.

Local Measures. Proposition 1 established concordance orders global regression slopes. Concordance not only orders standard regression estimates, but also the entire conditional linear relationship between generations and, therefore, local measures of mobility for specific subgroups. Increased concordance rotates or tilts the entire non-parametric regression curve; lowering mobility measured by the conditional expected rank $E\left[R^K \mid R^P = r\right]$.[13] I formalise this in the following proposition.

PROPOSITION 2 (Conditional Expected Rank Measures). *Economies A and B have identical marginals, but different rank dependence denoted by copulas C^A and C^B, respectively. For monotone conditional expected ranks, if $C^A \succeq C^B$ the conditional expected rank measure rotates around a point $r^\star$:*

i. It is decreasing relative to the concordance ordering below $r^\star$, $E^A\left[R^K \mid R^P = r\right] \leq E^B\left[R^K \mid R^P = r\right] \ \forall\, r \leq r^\star$, and

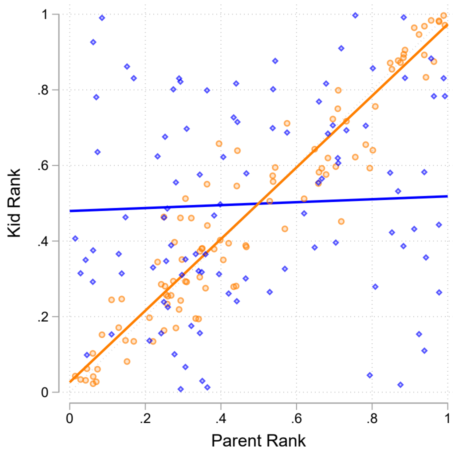
ii. It is increasing relative to the concordance ordering above $r^\star$, $E^A\left[R^K \mid R^P = r\right] \geq E^B\left[R^K \mid R^P = r\right] \ \forall\, r \geq r^\star$.

When the conditional expected rank is approximated parametricaly using the linear projection in equation (3), then $r^\star = 0.5$ and the conditional expected rank curve rotates around the median.

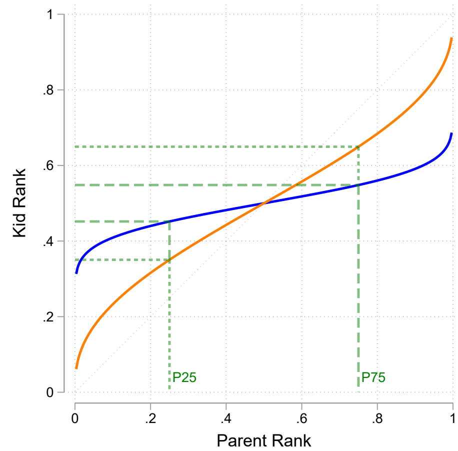
Higher concordance rotates the conditional expectation function, $E\left[R^K \mid R^P = r\right]$, toward the 45-degree line (Figure 2): children's expected rank responds more strongly to parental rank. Low-rank parents' children fall further below the median, high-rank parents' children rise further above. At perfect concordance $E\left[R^K \mid R^P = r\right] = r$ exactly as children's ranks coincide with their parents' and relative mobility vanishes.

[12] Hart (1983) and Shorrocks (1993) propose using one minus the intergenerational correlation as a measure of mobility. Similarly, increased concordance also raises *average* non-linear persistence measures (e.g., Arellano, Blundell, and Bonhomme 2017). The logic of the argument applies in one direction because not all economies can be compared by their concordance (see Section 5). However, if we restrict attention to those that can be ranked by concordance we can invert the logic. In this case (i) and (ii) must imply $C^A \succeq C^B$ and, likewise, the remaining results.

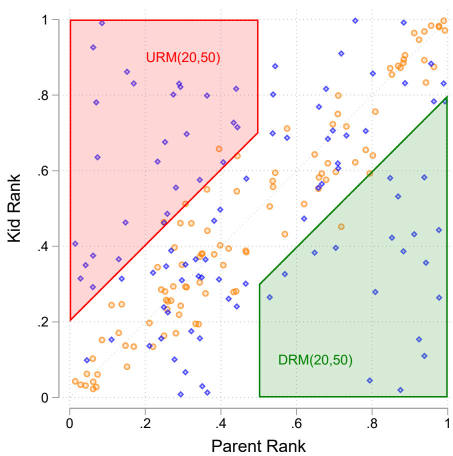
[13] See Chetty et al. (2026) for a recent application using this to measure mobility.



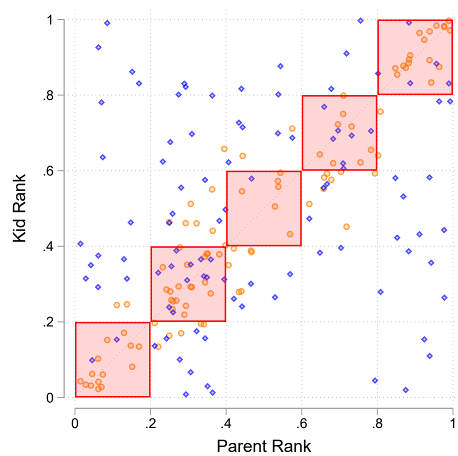
(A) Rank-Rank Regressions



(B) Conditional Expected Rank



(C) Directional Mobility



(D) Shorrocks (1978) Trace Measure

FIGURE 2. Mobility Measures Under Increased Concordance

Notes: Each panel shows the effect of an increase in concordance on a mobility measure. The marginal distribution of parents income and children's income are held fixed. In each panel, the correlation parameter in the elliptical copula increase and the joint-distribution of the orange circle markers is larger than the blue diamonds in the concordance order. Panel (A) corresponds to Proposition 1, with changing rank-rank correlation showing 100 draws from the joint distribution and the population value of the mobility measure. Panel (B) shows the population value of the CER rotates corresponding to Proposition 2. Dashed green lines correspond to the measure evaluated at the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentile which are commonly used in the literature. Panel (C) shows downwards rank mobility (in green) and upwards rank mobility (in red) as regions of the rank-space using above and below median parents who have children jumping more than a quintile in the child distribution. Panel (D) shows the Shorrocks trace measure in red as regions of the rank-space using quintiles of the distribution.

3.2. Transition Probabilities

While regression measures summarize mobility globally, practitioners often focus on specific transitions like "rags-to-riches" or "riches-to-rags" probabilities (e.g., Corak and Heisz 1999; Chetty et al. 2014a) which capture mobility for particular subgroups. Two of the results below—directional rank mobility and, later, the Shorrocks trace (Proposition 4iv)—require a stronger regularity condition than concordance alone.[14]

ASSUMPTION 1 (Anti-Diagonal to Diagonal Exchange in Mass). *Let $\Delta C(u,v) \equiv C^A(u,v) - C^B(u,v)$ denote the difference in the copulas A and B such that $A \succeq B$. $\Delta C(u,v)$ is a supermodular function: for all $u_1 \leq u_2, v_1 \leq v_2 \in [0,1]$*

$$\Delta C(u_1,v_1) + \Delta C(u_2,v_2) \geq \Delta C(u_2,v_1) + \Delta C(u_1,v_2).$$

This requires mass to shift toward the diagonal consistently throughout rank-space—similar ranks co-occur more often everywhere, not just on average—ruling out offsetting movements where concordance increases in some regions but decreases in others. For many parametric copula families, concordance is identical to Assumption 1.

PROPOSITION 3 (Rank Based Local Measures). *Economies A and B have identical marginals, but different rank dependence denoted by copulas C^A and C^B, respectively. If $C^A \succeq C^B$, then*

i. Transition Probabilities: Measures along the positive diagonal are increasing relative to the concordance ordering, while those in off diagonals are decreasing:

a. Positive diagonal: $TP^A\left[R^K > \tau^k \mid R^P > \tau^p\right] \geq TP^B\left[R^K > \tau^k \mid R^P > \tau^p\right]$ *and* $TP^A\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right] \geq TP^B\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right]$ $\forall \tau^k, \tau^p \in [0,1]$.

b. Off-diagonal: $TP^A\left[R^K > \tau^k \mid R^P \leq \tau^p\right] \leq TP^B\left[R^K > \tau^k \mid R^P \leq \tau^p\right]$ *and* $TP^A\left[R^K \leq \tau^k \mid R^P > \tau^p\right] \leq TP^B\left[R^K \leq \tau^k \mid R^P > \tau^p\right]$ $\forall \tau^k, \tau^p \in [0,1]$.

ii. Bhattacharya and Mazumder (2011)'s Directional Rank Mobility: If Assumption 1 holds then the probability a child's rank is larger than their parent's rank, by an amount s, for those with parents below τ is decreasing relative to the concordance ordering:

$$URM^B(s,\tau) \geq URM^A(s,\tau)\, \forall (s,\tau) \in [0,1]^2 \text{ where } URM(s,\tau) = Pr(R^K - R^P > s \mid R^P \leq \tau)$$

[14]This stronger assumption immediately implies concordance must be satisfied (see Appendix B). Imposing restrictions on the joint density is standard in mobility research. Chetty et al. (2017) and Berman (2022) invoke empirical plausibility arguments that implicitly require similar conditions. Shorrocks (1978) assumes away "*empirically unlikely*" transition matrices without quasi-maximal diagonals.

Analogously, Downward Rank Mobility (DRM).

Proposition 3 shows that under concordance all diagonal transitions increase, all off-diagonal transitions decrease. Unlike transition probabilities which depend only on copula values at specific points, directional mobility integrates over regions (see Figure 2 Panel C) and requires the stronger regularity condition that mass shifts uniformly toward the diagonal (Assumption 1).

This highlights the power of concordance as a unifying notion of dependence and organising framework. Across a range of local and global mobility notions, higher concordance uniformly implies lower mobility.

3.3. Fixed Rank Comparisons for Subpopulations

Empirical studies often evaluate ranks against a fixed external benchmark rather than within their own population. Examples include ranking children by their position in the parent cohort's income distribution; comparing subnational mobility against the national income distribution (e.g. Chetty et al. 2014a; Bütikofer, Dalla-Zuanna, and Salvanes 2022; Deutscher and Mazumder 2020; Bell, Blundell, and Machin 2023); or comparing outcomes across racial groups (in the spirit of Bayer and Charles 2018).[15] Let

$$\widetilde{R}^K = F_{\widetilde{Y}}\left(F^{-1}_{Y^K}\left(R^K\right)\right), \tag{4}$$

be the child's rank in a fixed reference distribution $\widetilde{Y}$, e.g. their ranking in the parent's income distribution or national distribution. Since this is strictly increasing in R^K it preserves the copula. Consequently, all rank-based exchange mobility measures remain monotone in concordance even under fixed benchmark rankings (an analogous argument applies for parent ranks). Those with higher income ranks in their own distribution are also higher in the fixed reference distribution of incomes.

The results so far establish concordance as sufficient for comparing mobility using various empirical approaches. I now turn to axiomatic measures, which take a different methodological approach by deriving mobility indices from first principles rather than statistical relationships.

[15] Nonparametric estimation of these fixed-benchmark ranks corresponds to absolute mobility measures (Deutscher and Mazumder 2023).

3.4. Axiomatic Measures of Exchange Mobility

A complementary approach axiomatizes mobility measures (similarly to inequality measurement). They precisely define minimal and maximal mobility as well as necessary properties of mobility measures. Axiomatic frameworks (e.g. Fields and Ok 1996; Cowell and Flachaire 2018) specify distance metrics on parent–child distributions that uniquely define a mobility index. Likewise, D'Agostino and Dardanoni (2009) derive a rank-based concordance index from ordering axioms, and Shorrocks' (1978) trace measure captures the probability of escaping one's parental "class". Under common marginals each of these indices hinges solely on the underlying dependence structure. Therefore, they all increase in the concordance order.

PROPOSITION 4 (Axiomatic Measures). *The following axiomatic measures of exchange mobility are decreasing in the concordance order:*

i. *Fields and Ok (1996, 1999)'s measures of absolute differences* $\int\int |Y^K - Y^P| f\left(Y^K, Y^P\right) dY^K dY^P$ *and it's decomposition into transitions.*

ii. *D'Agostino and Dardanoni (2009)'s concordance measure of rank mobility for global matrices and the extension .*

iii. *Cowell and Flachaire (2018)'s measure of the distance between relative positions,* $\frac{1}{\alpha(\alpha-1)} \int\int \left(\frac{Y^K}{\overline{Y}^K}\right)^{\alpha} \left(\frac{Y^P}{\overline{Y}^P}\right)^{1-\alpha} f\left(Y^K, Y^P\right) dY^K dY^P$, *where* α *controls the relative weight on upwards movements compared to downwards movements.*

Additionally, if Assumption 1 holds then

iv. *Shorrocks (1978)'s trace measure,* $\frac{q - trace(\mathcal{Q})}{q-1}$, *for stochastic matrix* $\mathcal{Q}$ *with elements* $\mathcal{Q}_{i,j} = Pr\left(\frac{j-1}{q} < R^K \leq \frac{j}{q} \mid \frac{i-1}{q} < R^P \leq \frac{i}{q}\right)$.[16]

These axiomatic results strengthen the case for concordance by showing it orders measures derived from different methodological traditions.

3.5. Proxies and Measurement Error

In practice, researchers often observe proxies—education, occupation, health status—rather than lifetime income. I show that the concordance order is maintained when using proxies.[17]

[16] This is obtained by discretizing ranks into q groups. For simplicity, I consider the case where q groups are of equal size, but this is not integral to the result.

[17] Defining proxies as functions of ranks is without loss of generality under common marginals.

Proposition 5 (Concordance of proxies). *Let a proxy E be determined by the following production function*

$$E^K = H^K\left(R^K, \varepsilon^K\right), \qquad E^P = H^P\left(R^P, \varepsilon^P\right).$$

Assume $H^x(\cdot,\cdot)$ are weakly increasing function for $x \in \{K, P\}$ and $(\varepsilon^K, \varepsilon^P)$ is independent of the pair R^K, R^P with a joint-distribution that is the same in economy A and economy B. Then if $C^A \succeq C^B$ the copula of the pair (E^K, E^P) satisfies $\widetilde{C}_E^A\left(R_E^K, R_E^P\right) \succeq \widetilde{C}_E^B\left(R_E^K, R_E^P\right)$.

Directly modeling dependence via copulas accommodates a variety of empirically relevant proxy frameworks, e.g. in education. In particular, it captures cases where education proxies income and where latent ability drives both education and income:

Example 1: Human capital determines incomes. Suppose increased human capital directly increases lifetime incomes (Mincer 1974) and we have $Y = H(E) + \varepsilon$. Inverting yields $E = H^{-1}(Y - \varepsilon)$. Treating ε as a vector allows for a general functional form for H and Y which can accommodate cases where the variance of the income residual increases in education as is standard in empirical work.

Example 2: Ability determines education and income. Alternatively, let a latent skill S determine both education and income: $E = H_E(S, \varepsilon)$ and $Y = H_Y(S, \varepsilon)$.

In each example the function H can differ across generations. Parent-child joint distributions, e.g. $F(Y^K, E^P)$ or $F(E^K, E^P)$, inherit the dependence structure of the copula $C(R^K, R^P)$, so concordance results extend immediately.

3.5.1. (Non-Classical) Measurement Error

Although noise attenuates measured dependence, concordance orderings are preserved under general measurement error—mismeasured incomes act as proxies in the sense of Proposition 5. Thus, even though estimates like rank–rank slopes or elasticities may be biased in level, their ordering across economies is invariant to measurement error.

Corollary 1 (Concordance under mismeasurement). *Let $(\varepsilon^K, \varepsilon^P) \perp (R^K, R^P)$ denote a vector of measurement errors that are independent of true incomes or ranks with $\varepsilon^K \perp \varepsilon^P$. Reported, or observed, incomes $(\widetilde{Y}^K, \widetilde{Y}^P)$ are increasing functions of true lifetime incomes (or ranks) and the measurement error. Therefore, they satisfy, the following measurement* *equations*

$$\widetilde{Y}^K = G_k(R^K, \varepsilon^K), \qquad \widetilde{Y}^P = G_p(R^P, \varepsilon^P),$$

for weakly increasing functions G_K and G_P.

Holding measurement equations and the distribution of measurement errors constant (i.e. the noise structure is the same in A and B), if $C^A \succeq C^B$ then $\widetilde{C}^A \succeq \widetilde{C}^B$, where $\widetilde{C}(u, v)$ denotes the copula for the reported or observed outcomes $(\widetilde{Y}^K, \widetilde{Y}^P)$.

Importantly, assuming observed measures are monotone functions of true earnings does not rule out non-classical measurement error (in the spirit of Bound et al. 1994) and standard noise structures satisfy Corollary 1 directly. Classical additive error in income levels, $\widetilde{Y}^s = Y^s + \varepsilon^s$, and error-in-ranks specifications of the form $\widetilde{R}^s = F_{\widetilde{s}}\left(F_s^{-1}(R^s) + \varepsilon^s\right)$ (Chetverikov and Wilhelm 2023) both yield observed measures that are weakly increasing in true lifetime income, satisfying the monotonicity requirement with generation-specific error variances held equal across economies.

Life-cycle biases. Systematic variation in the link between current and lifetime earnings (Jenkins 1987; Nybom and Stuhler 2016) produces:

$$\widetilde{Y}^K = G_K\left(F_K^{-1}(R^K) + \mu_{\varepsilon^K} + \sigma_{\varepsilon^K}\varepsilon^K\right), \qquad \widetilde{Y}^P = G_P\left(F_P^{-1}(R^P) + \mu_{\varepsilon^P} + \sigma_{\varepsilon^P}\varepsilon^P\right),$$

which also satisfies the Corollary's restrictions. Concordance orderings are therefore preserved even when estimates of the IGE or rank–rank slope are biased in levels by life-cycle effects. Similar points are made by Bhattacharya and Mazumder (2011) in the context of transition probabilities and Kitagawa, Nybom, and Stuhler (2018) in the context of rank-rank correlations, who also derive results for generalised life-cycle bias measurement equations.

3.6. Summary

Table 1 synthesizes the results, organizing measures by methodological approach. Columns summarise robustness to mass points, measurement error, and proxies.

TABLE 1. Which Measures Agree Under Concordance? A Practical Reference

	Properties		Concordance Ordering				
	Copula only?	Distribution of what?	Increasing?	Mass points?	Fixed ranks?	Proxies or measurement error?	Common marginals only?
Regression or Correlation measures							
IGE	No	Log income	Yes	Yes	N/A	Yes	Yes
IGC (Pearson correlation)	No	Log income	Yes	Yes	N/A	Yes	Yes
Rank–rank correlation	Yes	Ranks	Yes	Yes	Yes	Yes	No
CER	Yes	Ranks	Yes	Parametric Only	Yes	Yes	No
Transition matrix measures							
TP	Yes	Ranks	Yes	Yes	Yes	Yes	No
URM/DRM	Yes	Ranks	Yes[a]	No	Yes	Yes	No
Axiomatic measures							
Shorrocks	Yes	Ranks	Yes[a]	Yes	Yes	Yes	No
Fields & Ok	No	Income levels	Yes	Yes	N/A	Yes	No[b]
D'Agostino & Dardanoni	Yes	Ranks	Yes	Yes	Yes	Yes	No
Cowell & Flachaire	User choice	Income levels or ranks	Yes	Yes	Yes	Yes	No[b]

Notes: [a] refers to results that additionally require Assumption 1. Most results require only concordance (Definition 1). Two results—directional rank mobility (Proposition 3ii) and Shorrocks index (Proposition 4iv)—require the stronger Assumption. [b] denotes that common marginals are required unless the measure is evaluated using ranks or marginals also satisfy the usual stochastic order. Results that rule out atoms in column 4 also require continuous proxies or measurement error. Results summarize propositions in Sections 3 and 4. Proofs of the results are given in Appendix C. A 'Yes' in the 'Increasing?' column means the measure unambiguously increases (or decreases) with concordance when economies satisfy Definition 1. Applied researchers can use any measure with 'Yes' interchangeably.

4. Decomposing the Roles of Dependence and Inequality

Previous results assumed common marginals to isolate variation in the dependence structure alone. This corresponds to comparing hypothetical variants of a single economy. Comparisons across space and time—such as asking whether the U.S. is more mobile today than in 1965 (Chetty et al. 2017) or whether the U.S. is more mobile than Canada (Corak 2020)—typically involve different marginal income distributions. I now show that any change in a mobility measure can be decomposed into a component due to changing marginals and a component due to changing dependence.

Decomposition. By Sklar's theorem, any joint distribution can be written $F(y^K, y^P) = C\left(F_K(y^K), F_P(y^P)\right)$, where C is the copula and F_K, F_P are the marginal distributions. A mobility measure M therefore depends on both components: $M = M(C, F_K, F_P)$. For two economies A and B, the change in any measure can be decomposed as:

$$\begin{aligned}\Delta M &= M^A - M^B = M(C_B, F_K^B, F_P^B) - M(C_A, F_K^A, F_P^A) \\ &= \underbrace{\left[M(C_B, F_K^B, F_P^B) - M(C_A, F_K^B, F_P^B)\right]}_{\text{Copula component (varying dependence)}} + \underbrace{\left[M(C_A, F_K^B, F_P^B) - M(C_A, F_K^A, F_P^A)\right]}_{\text{Marginal component (varying marginals)}} . \end{aligned} \tag{5}$$

The first term captures the change in mobility due to differences in the dependence structure, holding marginals fixed at economy B's distributions. The second term captures the change due to differences in marginals, holding the copula fixed at economy A's dependence structure. An alternative decomposition holds marginals at A and the copula at B, averaging yields:

$$\begin{aligned}\Delta M = &\underbrace{\left[\tfrac{1}{2}\Big(M(C_B, F_K^B, F_P^B) - M(C_A, F_K^B, F_P^B)\Big) + \tfrac{1}{2}\Big(M(C_B, F_K^A, F_P^A) - M(C_A, F_K^A, F_P^A)\Big)\right]}_{\text{Copula component}} \\ &+ \underbrace{\left[\tfrac{1}{2}\Big(M(C_A, F_K^B, F_P^B) - M(C_A, F_K^A, F_P^A)\Big) + \tfrac{1}{2}\Big(M(C_B, F_K^B, F_P^B) - M(C_B, F_K^A, F_P^A)\Big)\right]}_{\text{Marginal component}} . \end{aligned} \tag{6}$$

This decomposition clarifies when a concordance ordering applies. For the copula component, holding marginals fixed, the results of Section 3 apply: if $C^B \succ C^A$, all measures in Table 1 are ordered. For the marginal component, which holds the copula fixed, all rank-based measures are invariant—since ranks have uniform marginals by construction, changes in income distributions leave the rank-rank correlation, transition matrices, and Shorrocks index unchanged. Level-based measures, however, respond to marginal shifts even when dependence is unchanged. Appendix D illustrates the decomposition in closed form using the bivariate log-normal, showing that the rank-rank correlation depends only on the correlation parameter ρ while the IGE depends on $\rho \cdot \sigma_K/\sigma_P$, so rising inequality can reverse the IGE's trend even as rank-dependence weakens.

Implications. Rank-based measures isolate "pure" dependence: their marginal component is zero by construction, so they are ordered whenever copulas are ordered, regardless of differences in income distributions across economies. Proxy-based indices mapping ranks to observed categories (e.g., social classes to occupations) are similarly robust.[18] Rank-based measures are thus ideal for cross-country comparisons using published estimates as their marginal component is zero by construction, so comparing rank-rank slopes across countries with different inequality levels is equivalent to comparing copulas directly.

In contrast, level-based measures (i.e., the IGE or IGC) conflate both sources of variation. Thus, even when the copula component is ordered by concordance, the marginal component can obscure or reverse the overall ranking.[19] Column 6 of Table 1 summarizes these limits.

Note that this also clarifies debates about which income concept to use. For example, if consumption better reflects permanent income, consumption-rank and earnings-rank mobility measures will agree when copulas are concordant, but general consumption-based or earnings-based measures may diverge when earnings inequality changes faster than consumption inequality. The decomposition in this section offers a principled approach to understand these disagreements.

5. When Concordance Fails

Sections 2 and 3 establish that concordance provides a powerful unifying framework: when two economies can be ranked by their copulas, all standard mobility measures agree on which is more mobile. However, concordance is only a *partial* order—not all pairs of economies can be compared. This section examines when concordance fails and why different mobility measures then disagree. Section 6 addresses how researchers should proceed in such cases.

5.1. The Limits of Concordance

When can concordance fail to rank two economies? Figure 3 illustrates this directly. Within each column, transition matrices increase in concordance. However, no

[18] This implies rank-based alternative versions of measures are ordered. For example, the Fields and Ok (1996) measure computed using ranks corresponds to the Bartholomew (1973) average jump measure. The assumption that proxies depend on ranks, not levels, can be tested in auxiliary datasets without information on two generations.

[19] Under stochastic dominance of the marginal distributions, a general ranking for all supermodular measures obtains (see Meyer and Strulovici 2013). When marginals are not identical but are sufficiently close, continuity results apply analogously to Proposition 6.

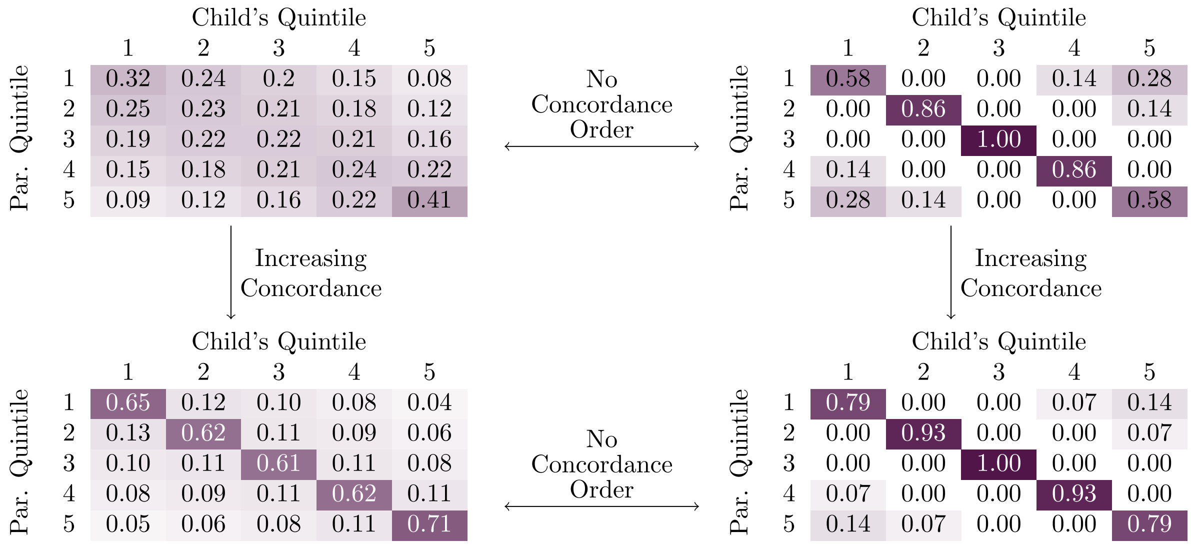


FIGURE 3. An Example of Concordance as a Partial Order

Notes: The first row is adapted from an example in Berman (2022). Each row produces numerically equivalent values for the rank correlation.

concordance ordering exists *between* columns or within rows. The matrices in each row produce numerically identical rank correlations (0.32 and 0.65, respectively), yet differ substantially in their transition probabilities or mass along the trace. This shows that a single global measure does not determine the entire copula when concordance fails.[20]

5.2. Why Mobility Measures Disagree

When concordance doesn't order distributions, why do different mobility measures produce conflicting rankings? Each measure weights different regions of the joint distribution and any mobility measure can be expressed as a weighted integral of the joint density

$$M_M = \int\int \phi_M\left(Y^K, Y^P\right) f\left(Y^K, Y^P\right) dY^K dY^P, \tag{7}$$

where $\phi_M\left(Y^K, Y^P\right)$ is a mobility-measure specific intensity assigning importance to parent-child income pairs. When comparing economies A and B with different copulas but common marginals, the difference in any twice-differentiable measure can be

[20] Berman (2022) argues copulas in the left column are empirically plausible (comparing to estimates in Jäntti et al. 2006 and Chetty et al. 2017), but rejects the right-column "all-or-nothing" dynamics. This reinforces the sense that the concordance restriction is implicit in applied work.

decomposed as:

$$M_M^A - M_M^B = \int_0^1 \int_0^1 \underbrace{\frac{\phi_{M,12}\left(F_K^{-1}(R^K), F_P^{-1}(R^P)\right)}{f_K\left(F_K^{-1}(R^K)\right)\ f_P\left(F_P^{-1}(R^P)\right)}}_{W_M(R^K,R^P)} \left[C^A(R^K, R^P) - C^B(R^K, R^P)\right]\, dR^K\, dR^P. \tag{8}$$

When concordance orders economies, $C^A(R^K, R^P) - C^B(R^K, R^P) \geq 0$ everywhere in equation (8), so all measures agree despite different weights W_M. When concordance fails, the difference in copulas changes sign across regions. In this case, one measure may place high weight on regions where $C^A > C^B$ while another emphasises regions where $C^A < C^B$, producing conflicting rankings.

This explains the different levels of sensitivity of measures to concordance violations. Some measures average dependence broadly across the joint distribution. The rank-rank slope, for instance, has an intensity that assigns uniform weight $W_\rho = 12$ everywhere—it is, in effect, an unweighted average of ΔC over the unit square. Such measures are relatively insensitive to local concordance violations because positive and negative regions can offset. Other measures zoom in on specific regions. Transition probabilities at particular thresholds have indicator-function intensities—they depend on ΔC at a single point, with zero weight elsewhere. These measures are maximally sensitive to which side of a sign change they happen to evaluate. The intergenerational elasticity falls between these extremes: its intensity loads most heavily on the tails (families furthest from the mean, $\phi_M\left(Y^K, Y^P\right) = [\ln Y^K - \overline{\ln Y^K}][\ln Y^P - \overline{\ln Y^P}]$), making it sensitive to concordance violations concentrated in those regions.

Measure choice then embeds an implicit judgment over what constitutes desirable or relevant mobility (Fields and Ok 1996; Jäntti and Jenkins 2015; Ray and Genicot 2023), similar to selecting a social welfare function when Lorenz curves intersect (Atkinson 1970).

5.3. Disagreement is Generic

Inspecting equation (8) shows that it is always feasible to construct two measures that disagree when concordance fails. But concordance provides a much sharper characterisation of disagreement among *widely used* measures.

First, concordance is not just sufficient for agreement—it is necessary. It is required for transition probabilities to order mobility consistently at all cutpoints the data can support, a direct consequence of the if-and-only-if characterisation in Proposition 3. Second, even for pre-specified discretisations (e.g., quintile, decile, or ventile transition matrices) concordance violations manifest in ranking reversals among standard empirical objects.

Corollary 2 (Ranking Reversals at Standard Discretisations). *Let C^A and C^B be copulas with densities c^A and c^B such that $\Delta C \equiv C^A - C^B$ changes sign. The conditions below can equivalently be stated in terms of the copula measure of ΔC, but the density formulation is more transparent. Write $\Delta c \equiv c^A - c^B$. Fix two reporting grids*

$$\Pi_1 = \{0 = t_0^1 < t_1^1 < \cdots < t_K^1 = 1\}, \quad \Pi_2 = \{0 = t_0^2 < t_1^2 < \cdots < t_J^2 = 1\}$$

e.g., quintiles and deciles. Define cells $R^1_{ij} = [t^1_{i-1}, t^1_i] \times [t^1_{j-1}, t^1_j]$ and $R^2_{kl} = [t^2_{k-1}, t^2_k] \times [t^2_{l-1}, t^2_l]$.

If there exist cells such that:

i. *Δc maintains constant sign on each cell, and*

ii. *$\int\int_{R^1_{i,j}} \Delta c(u, v)\, du\, dv > 0$, but $\int\int_{R^2_{k,l}} \Delta c(u, v)\, du\, dv < 0$,*

then transition probabilities reverse rankings: Grid Π_1 ranks A as less mobile, but Π_2 ranks A as more mobile. When both detecting cells lie on the diagonal ($i = j$ and $k = l$), the Shorrocks trace measure can also reverse ranking, provided the detecting cells are not outweighed by offsetting contributions from the remaining diagonal cells.

When concordance holds, all standard measures agree. When it fails, some must disagree. Researchers cannot avoid these disagreements by restricting attention to "simpler" or more "transparent" measures as Corollary 2 shows disagreement appears in the transition matrices routinely published in empirical work. A researcher using quintile transition matrices may conclude economy A has lower mobility, while another using decile matrices concludes A has higher mobility. This is not measurement error, but reflects genuine ambiguity in how to weight regions where parent-child dependence differs in opposite directions. Whether concordance holds between empirical copulas can be tested directly using moment inequality methods; Section 7 implements such a test and Appendix J provides details.

6. What To Do When Concordance Fails

The previous section established that concordance failures produce genuine, unavoidable disagreements among standard mobility measures. This section provides practical guidance for researchers: when violations are small, how much do they matter (6.1)? when they are substantive, what should be reported (6.2)? and when a single ranking is needed, how should the partial order be completed (6.3)?

6.1. Approximate Concordance

Concordance results are not a knife-edge property. If two copulas nearly satisfy concordance, with small violations, measures are approximately ordered.

PROPOSITION 6 (Continuity and Approximate Concordance). *Let M be any real-valued function of the copula C (with fixed marginals) such that*

i. (Concordance Monotonicity) for any copulas C', C'', if $C' \succeq C''$ then $M(C') \leq M(C'')$

ii. (Uniform Continuity) M is uniformly continuous with respect to the copula sup-norm $||C' - C''||_\infty \equiv \sup_{(u,v)\in[0,1]^2}$.

If there exists a copula $C^\dagger$ such that $C^A \succeq C^\dagger$ and $||C^B - C^\dagger||_\infty \leq \epsilon$ (i.e., C^B is obtained as an ϵ-perturbation of a copula $C^\dagger$ that is dominated by C^A in the concordance order), then

$$M(C^A) \leq M(C^B) + \omega_M(\epsilon)\,, \tag{9}$$

where $\omega_M(\epsilon) \equiv \sup\{|M(C') - M(C'')| : ||C' - C''||_\infty \leq \epsilon\}$ is the measure M's modulus of continuity. In particular, if M is Lipschitz continuous under the copula sup-norm ($||\cdot||_\infty$) with constant L_M, then

$$M(C^A) \leq M(C^B) + L_M\epsilon\,. \tag{10}$$

The bound L_M varies across measures and determines their sensitivity to concordance violations. Measures that average dependence broadly—such as the rank-rank slope—degrade smoothly under small violations, while measures that concentrate on narrow regions are more sensitive. This explains the tighter co-movement among rank-based measures observed in empirical work (Deutscher and Mazumder 2023). Appendix F derives L_M for measures in Table 1 and discusses measure selection motivated by robustness to approximate concordance.

6.2. Reporting When Concordance Fails

When concordance fails substantively—copulas exhibit non-trivial offsetting changes across regions—reporting only a single global measure is inadequate. For example, researchers using different discretisations can reach opposite conclusions about the same data. The intensity decomposition in equation (8) shows a single global measure embeds a particular weighting of regions where ΔC takes opposite signs, and different readers' preferred measures embed different weightings.

This problem has an intuitive solution: report familiar estimands—rank-rank slopes or transition probabilities—computed locally in the regions where concordance holds and fails. This allows each reader to evaluate whether the concordance violation occurs in a region that matters for their preferred measure.[21]

The concordance test itself provides the necessary guidance on *where* to report. The moment inequality test in Section 7 evaluates ΔC on a grid of rank-space. The grid points where violations are detected—where $\hat{C}^A(u, v) < \hat{C}^B(u, v)$—identify the regions where dependence shifted in the direction opposite to the global trend. These regions define a natural partition for local reporting.

A reporting protocol. Concretely, the following protocol ensures that readers with different preferred measures can assess how concordance violations affect their conclusions.

(a) *Test concordance*. Apply the moment inequality test (Section 7) on a grid of rank-space, testing both $C^A \succeq C^B$ and $C^B \succeq C^A$.

(b) *If concordance holds*: report any preferred global measure and note that all concordance-monotone measures agree on the ranking. A single statistic suffices.

(c) *If concordance fails*:

  (a) Identify the regions of rank-space where ΔC changes sign, using the grid-level test output.

  (b) Report *local rank-rank slopes* within each region. Local slopes treat all parts of each region equally—they do not privilege one reader's preferred cutpoint over another's—making them a neutral local summary.[22]

[21]This reporting strategy can be formalised as a communication-admissibility requirement in the sense of Andrews and Shapiro (2021) for users with different preferred measures; see Appendix G for details.

[22]Formally, the rank-rank slope's intensity kernel is constant ($W_\rho = 12$ everywhere), so its local version provides an unweighted average of ΔC within each region. For any reader whose preferred measure has a smooth intensity, local slopes approximate their measure's local contribution up to an error that shrinks as the partition refines (see Appendix F).

(c) Supplement with *transition probabilities* at standard thresholds (e.g., quintile boundaries) for readers who focus on specific mobility pathways. Transition probabilities depend on ΔC at specific points rather than averages, so they serve readers whose preferred measures have concentrated rather than smooth intensities.

(d) *Optionally*: report the Lipschitz bounds from Appendix F so readers can assess the maximum effect of the concordance violation on their preferred measure.

The partition into regions need not be fine. Proposition 6 ensures that for any measure with Lipschitz constant L_M, the approximation error from a partition with J bins is of order L_M/J. For broadly-averaging measures (rank-rank slopes, intergenerational correlations), a coarse, data informed, partition captures most of the relevant heterogeneity. For concentrated measures (specific transition probabilities), the point evaluations in step (c) provide exact information.

When local slopes agree in sign across all regions, the violation is mild and Proposition 6 bounds the discrepancy; when they disagree in sign, the local slopes are themselves the substantive finding. Appendix F provides the corresponding Lipschitz constants to quantify the maximum effect of a violation on any given measure.

6.3. Completing the Concordance Order

When concordance definitively fails and a single ranking is nevertheless required—for example, to construct a time series or country ranking—researchers need a principled tie-breaking rule. I discuss two approaches.

Restricting the comparison set. The first approach limits attention to economies that can be ranked by concordance. Many parametric copula families naturally induce concordance orderings as dependence parameters vary (Joe 1997, ch 4). This directly rules out pathological shifts like those in Figure 3 and retains only comparisons for which all measures agree. The cost is loss of generality.

Axiomatic completion. The second approach constructs an explicit tie-breaking rule. D'Agostino and Dardanoni (2009) provide axiomatic foundations for completions in the context of discrete transition matrices, prioritising rank distance: mobility increases with the magnitude of rank changes, regardless of location. This yields the following minimum-distance completion.

Definition 3 (A Minimum Distance Completion of the Concordance Order). *For any empirical copula $C(u, v)$, construct the following summary statistic by mixing between the independent copula, $C^{\square}_{uv} = uv$, and the Hoeffding upper bound, $M(u, v) = \min(u, v)$:*

$$M_\lambda = \lambda^\star \in argmin_{\lambda \in [0,1]} \int_0^1 \int_0^1 \left[C(u, v) - ((1 - \lambda)\, M(u, v) + \lambda\, C^{\square}_{uv})\right]^2 du\, dv. \tag{11}$$

Intuitively, this asks what mixture of perfect immobility (Hoeffding bound M) and independence ($C^{\square}$) best approximates observed mobility? The weight on independence, $\lambda^\star$, provides the mobility index—higher values indicate more mobility.[23] The completion reduces to pure concordance comparisons when those exist and is used in related contexts. Fernández and Rogerson (2001) and Abbott et al. (2019) use an identical minimum-distance estimator to measure marital sorting. Chiappori et al. (2025) use an odds-ratio index axiomatised for assortative mating which is equivalent to concordance in discrete tables, providing alternative normative foundations. An alternative welfare-consistent completion, in which a planner with submodular preferences over parent-child incomes ranks economies, is discussed in Appendix H.

Connection to inequality measurement. Concordance is the analogue to Lorenz dominance for mobility.[24] The parallel extends to completions. Just as intersecting Lorenz curves require specifying a social welfare function to produce a complete ranking (Atkinson 1970), crossing copulas require analogous normative choices—or, alternatively, the local reporting protocol described above.

7. Revisiting Intergenerational Mobility Over the 20th Century

Having established theoretical conditions under which mobility measures agree, I now test these predictions using U.S. data across the 20th century.

To link the theoretical results explored above to the study of intergenerational mobility, I construct cohort-specific mobility estimates for U.S. males born between

[23] Using the first order condition, the value of this mobility measure is $M_\lambda = 1 - \int_0^1 \int_0^1 (C(u, v) - u\,v)\left(\min(u, v) - u\,v\right) du\, dv \Big/ \int_0^1 \int_0^1 (\min(u, v) - u\,v)^2\, du\, dv$.

[24] A similar connection was first proposed by Dardanoni (1993). Atkinson and Bourguignon (1982) highlight concordance-increasing exchanges in the context of multi-dimensional inequality: when social welfare is utilitarian and sub-modular at the level of parent-child incomes, increased concordance lowers both mobility and social welfare.

1910 and 1980. To limit life-cycle bias I restrict the sample to fathers and sons aged between 30 and 50. Following Jácome, Kuziemko, and Naidu (2025), I pool all available surveys that report respondents' current family income together with race, father's occupation, and region of birth or childhood and impute parental income from auxiliary data sources (primarily the Census) using race, occupation, and region.[25] This yields a repeated cross-section that is nationally representative and consistent over time which allows me to document trends in intergenerational mobility measures over the 20th century (see also, Davis and Mazumder 2024).

The theoretical framework makes a sharp prediction: when two cohorts can be ranked by concordance, all rank-based mobility measures must agree on which is more mobile, while level-based measures may diverge as marginal distributions change. I test this prediction directly. I first summarise the overall pattern using the cardinal concordance measure, then establish which cohort pairs satisfy the concordance order formally, and show that individual mobility measures move together exactly where concordance holds and exhibit the predicted divergence where it fails.

7.1. Concordance Mobility

The cardinal concordance measure from the completion in equation (11) provides a convenient summary of mobility across all cohorts before turning to the formal tests. Figure 4 plots this measure over the 20th century. Mobility increases from 0.66 to 0.78, with a wave-like pattern: rising through mid-century, plateauing for the 1940s–1950s cohorts, and declining modestly for the 1960s birth cohort before recovering.[26]

7.2. Testing the Concordance Order

To formally test the order, I construct a moment-inequality test following Andrews and Soares (2010). For any two cohorts A and B, concordance $C^A \succeq C^B$ requires $C^A(u, v) \geq C^B(u, v)$ at every point in the unit square. I evaluate this condition on a 10×10 grid of rank-space, forming the studentised moments $\hat{m}_g/\hat{\sigma}_g$ at each grid point g, and take the

[25] For linear estimating equations this is a Two-Sample Instrumental Variable design. Like many studies of mobility in a historical context (e.g., Collins and Wanamaker 2022; Ward 2023), this application uses both self-reported incomes and a proxy approach to construct linked parent-child outcomes. The theoretical characterisation of mobility measures shows that results are robust to using proxies or the presence of measurement error.

[26] This wave-like pattern—rising through mid-century, plateauing thereafter—likely reflects the interplay of expanding secondary education, compression of wage structures during the 1940s–70s, and subsequent increases in skill premia, though identifying specific mechanisms is beyond this paper's scope. See Davis and Mazumder (2024) for further discussion.

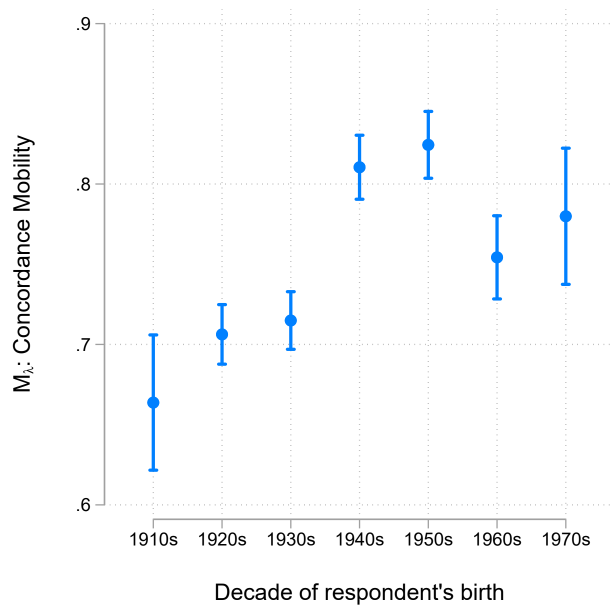


FIGURE 4. Concordance Mobility Over the 20$^{\text{th}}$ Century

Notes: Author's estimates from 15 combined US data sources (see text, Jácome, Kuziemko, and Naidu 2025 and Appendix I for details) by birth decade for respondents ages 30–50. Parental income predicted using family income conditional on father's occupation, race, and region (South vs. elsewhere) from auxiliary data (often the census) as close as possible to the respondents' tenth birthday (see Jácome, Kuziemko, and Naidu 2025 sec. III.B for more details). Sample weights are used and I reweight each birth cohort (i.e., decade) so that they have representative race×sex shares. Estimate of concordance mobility measure in (11) and 95% confidence interval from 400 bootstrap replications.

minimum as the test statistic. I test both directions ($A \succeq B$ and $B \succeq A$) and classify each cohort pair accordingly.[27]

Table 2 reports the results. Panel A tests long-run comparisons against the 1910s baseline. The results are striking: for every subsequent decade, I reject $C^B \succeq C^A$ at the 5% level while failing to reject $C^A \succeq C^B$, implying that the 1910s copula dominates all later cohorts in the concordance order. Intergenerational dependence was unambiguously stronger for the earliest cohort, and the long-run increase in mobility over the 20th century is robust to the choice of rank-based measure.

Panel B examines sequential decades, revealing a richer picture. Concordance orders four adjacent pairs. The 1910s–1920s, 1930s–1940s, and 1960s–1970s all show

[27] Full details of the testing procedure, including the choice of grid, critical values, and bootstrap implementation, are provided in Appendix J. The test is analogous to a Kolmogorov-Smirnov test for stochastic dominance, applied to the bivariate copula rather than the marginal distribution.

TABLE 2. Testing the Concordance Order Between Birth Cohorts

		p-values		
Decade A	Decade B	$H_0: C^A \succeq C^B$	$H_0: C^B \succeq C^A$	Interpretation
Panel A: Long-run comparisons (relative to 1910s)				
1910	1920	0.227	0.040^{*}	$C^A \succeq C^B$
1910	1930	0.296	0.001^{***}	$C^A \succeq C^B$
1910	1940	0.092	0.001^{***}	$C^A \succeq C^B$
1910	1950	0.708	0.001^{***}	$C^A \succeq C^B$
1910	1960	0.701	0.002^{**}	$C^A \succeq C^B$
1910	1970	0.470	0.001^{***}	$C^A \succeq C^B$
Panel B: Sequential decades				
1920	1930	0.001^{***}	0.004^{**}	Copulas cross
1930	1940	0.052	0.001^{***}	$C^A \succeq C^B$
1940	1950	0.142	0.358	Fail to reject
1950	1960	0.004^{**}	0.045^{*}	Copulas cross
1960	1970	0.190	0.017^{*}	$C^A \succeq C^B$

Notes: Moment-inequality tests of the concordance order following Andrews and Soares (2010) evaluated on a 10×10 grid of rank-space. The null hypothesis $H_0: C^A \succeq C^B$ tests whether the copula of the earlier cohort (Decade A) dominates that of the later cohort (Decade B) pointwise. "$C^A \succeq C^B$" indicates we reject $B \succeq A$ but fail to reject $A \succeq B$, consistent with declining concordance (rising mobility). "Copulas cross" indicates both directions are rejected and no concordance ordering exists. "Fail to reject" indicates neither direction is rejected. Bootstrap p-values based on 999 replications. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

declining concordance (rising mobility), and the 1940s–1950s comparison is indeterminate. However, two adjacent pairs—the 1920s–1930s and 1950s–1960s—exhibit crossing copulas, where the concordance order is rejected in both directions. For these pairs, dependence strengthened in some regions of the joint distribution while weakening in others. Theory predicts that mobility measures may disagree for exactly these cohort pairs, while agreeing where concordance orders pairs. I return to this prediction below.

The concordance test disciplines the interpretation of Figure 4. Where the partial order holds (Panel A), the cardinal summary faithfully reflects the underlying ordering. Where copulas cross (the 1920s–1930s and 1950s–1960s transitions in Panel B), the completion measure provides a useful but imperfect summary—small movements in

M_λ between these cohort pairs should be interpreted with caution, as different mobility measures need not agree on the direction of change.

7.3. Do Mobility Measures Move Together?

I now document trends in individual mobility measures and assess whether the patterns conform to the theoretical predictions. Figures 5 reports the intergenerational elasticity (IGE), the log-income correlation, the rank-rank slope, directional and trace-based measures from the transition matrix, and axiomatic mobility measures. I normalise all measures so that they increase with mobility (i.e., one minus persistence). Details of the implementation are deferred to Appendix I.

Panel (A) of Figure 5 replicates the broad patterns in Jácome, Kuziemko, and Naidu (2025):[28] rank persistence and the income correlation fall—thus mobility rises—through the mid-20th century and then stabilise, while the IGE rises as income dispersion widens in the latter half of the century. As established in Section 4, this divergence reflects the IGE's sensitivity to changing marginal dispersion.

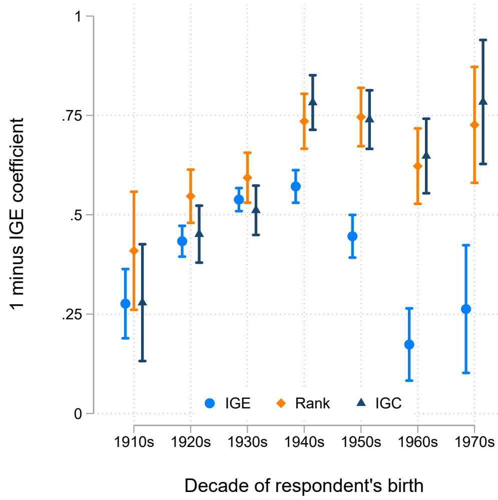
The concordance framework makes two testable predictions about the remaining measures. First, for cohort pairs where concordance holds (Table 2, Panel B), all rank-based measures should agree on the direction of change. Second, for cohort pairs where copulas cross, rank-based measures may disagree. Panels (B)–(D) of Figure 5 confirm both predictions. Directional mobility (Panel B), on-diagonal transition probabilities and the Shorrocks trace (Panel C), and off-diagonal transitions (Panel D) all display the wave-like pattern of the rank-rank slope. Where concordance holds—notably the long-run decline from the 1910s to any later cohort—all rank-based measures move in the same direction. The non-monotonic transitions between the 1920s–1930s and 1950s–1960s, where concordance tests detect crossing copulas, coincide with instances where individual measures exhibit significant deviations from the overall pattern.[29]

7.4. Decomposing the Evolution of Mobility

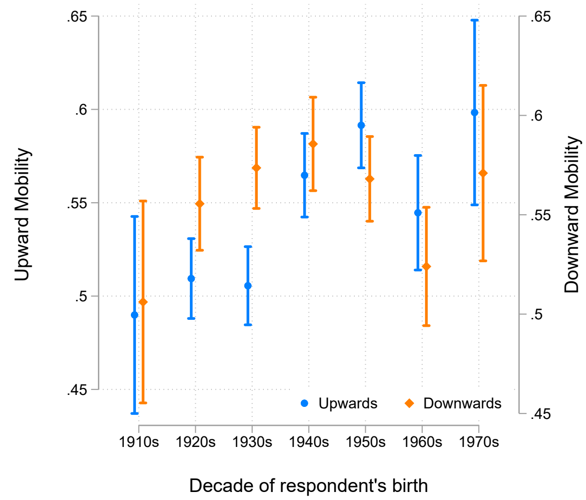
The decomposition in equation (6) allows me to quantify how much of the change in each measure reflects shifts in the dependence structure versus changes in marginal

[28] This replicates results in their Figure 1. However, due to differences in the underlying datasets (see Appendix I) this is best thought of as a distinct replication rather than a reproduction with modest quantitative differences; lending support to the overall validity of their results.

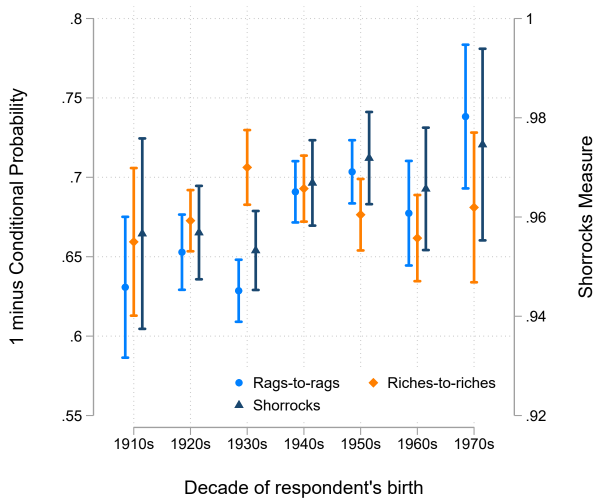
[29] For example, the ranking of the 1920s and 1930s cohorts by transition probabilities in Panels (C) and (D) shows disagreement—consistent with the crossing copulas detected in Table 2. Although disagreement can be driven by statistical uncertainty, in this case disagreement is statistically significant.



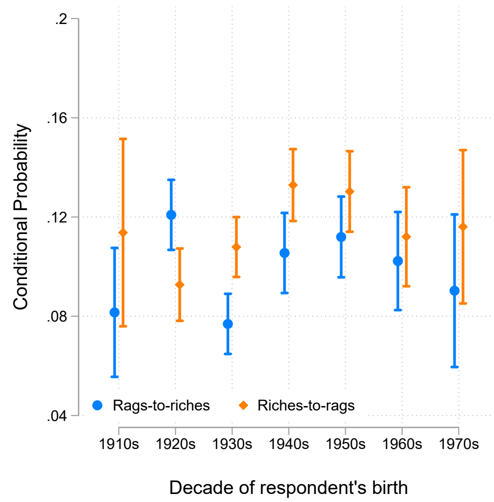
(A) Regression Estimates



(B) Directional Rank-Mobility



(C) Transition Probabilities (On-Diagonal)



(D) Transition Probabilities (Off-Diagonal)

FIGURE 5. Trends in Intergenerational Mobility Measures

Notes: See Figure 4 for data construction details. Panel (A) reports estimates of IGE, IGC, and rank-rank slope corresponding to Proposition 1. Panel (B) reports estimates of directional rank mobility measures using quintiles ($Pr(R^K - R^P > 0.2 \mid R^P \leq 0.2)$ and $Pr(R^K - R^P < -0.2 \mid R^P > 0.8)$) in Proposition 3. Panels (C) and (D) report transition probabilities using quintiles (Proposition 3) and panel (C) additionally reports the Shorrocks Trace Measure (Proposition 4) using deciles. Panels (B)-(D) use 400 bootstrap replications for inference. Panels (A) and (C) normalise measures to increase with mobility (one minus persistence).

distributions. Figure 6 reports results, highlighting why level-based measures diverge from rank-based measures in practice.

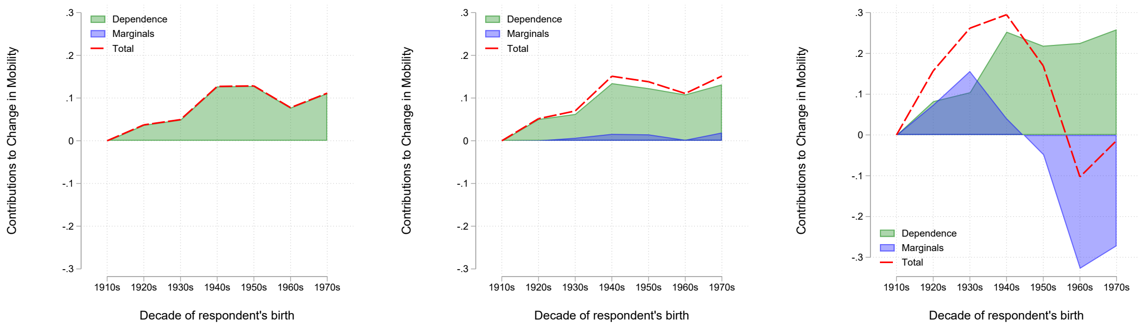
Panel (A) shows the decomposition for rank-rank mobility. For every rank-based measure, all variation comes from the dependence component—marginals are uniform by construction. The concordance tests in Table 2 confirm that this dependence component can itself be partially ordered: where concordance holds, the direction of change in the dependence component is unambiguous across all rank-based measures. Panels (B) and (C) reveal the source of divergence between the IGC and IGE documented in Figure 5. For the IGC, dependence and marginal components move together: both contribute to rising mobility through mid-century. For the IGE, however, the components move in opposite directions. The dependence component shows rising mobility throughout the century, consistent with rank-based measures, but is increasingly offset by the marginal component as inequality grows. By the 1970s cohort, rising inequality fully reverses the mobility gains from reduced dependence, producing the apparent decline in IGE-measured mobility.

These results offer practical guidance for empirical researchers. When marginal distributions are stable or when the research question concerns relative position, rank-based measures and the IGC provide clean estimates of changes in the intergenerational transmission mechanism. When marginals shift substantially—as occurs when comparing across countries with different inequality levels or across time periods spanning major distributional changes—the IGE will conflate changes in dependence with changes in dispersion. The decomposition provides a diagnostic. When the marginal component is large relative to the dependence component, researchers should either report rank-based measures or explicitly account for how changing inequality affects their chosen measure and conclusions. In this application, the marginal component of the IGE exceeds the dependence component by the 1960s cohort (Panel C), indicating that post-1950 trends in the IGE primarily reflect rising inequality rather than the changing intergenerational transmission of economic status.

Other Measures. Appendix Figure K.1 shows that Fields and Ok (1996) and Cowell and Flachaire (2018) measures display similar overall trends. Moreover, it shows that dependence-driven gains are dominated by changes in the marginal distribution.

8. A Structural Interpretation of the Concordance Order

The preceding sections showed that concordance holds for all long-run cohort comparisons and for most adjacent pairs, but fails for two—the 1920s–1930s and



(A) Rank-Rank Mobility (B) IGC (C) IGE

FIGURE 6. Decomposing Intergenerational Mobility Measures

Notes: See Figure 4 for data construction details. Each panel decomposes the change in mobility (relative to the 1910s cohort) into a dependence (copula) component and a marginal component following equation (6), with Shapley averaging over both orderings to address path dependence. Panel (A) reports the rank-rank slope (Proposition 1); the marginal component is zero by construction since rank marginals are uniform. Panel (B) reports the intergenerational correlation (IGC) and Panel (C) reports the intergenerational elasticity (IGE), both corresponding to Proposition 1.

1950s–1960s transitions, where copulas cross. What economic mechanisms generate a concordance order, and when should we expect it to fail? I show that both patterns arise naturally in Becker and Tomes (1979)–Loury (1981) models of human capital investment:[30] Broad-based expansions of investment produce concordance, while heterogeneous interventions that differentially affect families at different points in the income distribution can cause copulas to cross.

Consider a standard dynastic model. A family consists of a parent and child indexed by P and K. Parents value their own consumption and their child's consumption, with altruism parameter $\delta = 1/1{+}r$. They choose investment, I, in the future human capital of their child, solving

$$\max_I \; U\left(Y^P - I\right) + \delta\, \mathbb{E}_\theta\left[U\left(W\left(\theta f(I, H^P)\right)\right)\right], \tag{12}$$

where f is the skill-formation technology (weakly increasing and supermodular), W maps human capital to income, $\delta = 1/1{+}r$ is the altruism parameter, and θ is a luck shock

[30] I abstract from parental beliefs, dynamic complementarity, multidimensional skills, and borrowing constraints (see e.g., Cunha and Heckman 2007; Caucutt and Lochner 2020; Attanasio, Meghir, and Nix 2020; Caucutt et al. 2020; Moschini 2023).

realised after investment and independent of family choices.[31]

8.1. When Does the Concordance Order Arise?

Many economic forces increase investment uniformly across the parental income distribution: higher returns to skill (wage schedule W), better learning technology (production function f), or greater patience (discount factor δ). Lemma 1 shows that any such change increases concordance.

LEMMA 1 (Concordance and an outward expansion of investment). *Assume that the human capital production function $f(\cdot,\cdot)$ is weakly increasing and supermodular in both arguments. Then, holding the distribution of parental incomes fixed, any two economies A and B that produce the following pointwise relation on endogenous investment decisions $I^\star$,*

$$I^{\star A}\left(Y^P\right) \geq I^{\star B}\left(Y^P\right) \quad \forall\, Y^P, \tag{13}$$

also produce the following concordance order on parent-child incomes and their copulas:

$$A \succeq B \quad \textit{equivalently} \quad C^A \succeq C^B. \tag{14}$$

The following corollary links this directly to interpretable model primitives. The proof, along with extensions to a broader class of policy shifts, is in Appendix L.

COROLLARY 3 (Mechanisms producing increased concordance). *The following mechanisms all imply a pointwise order on investment policy functions, $I^{\star A}\left(Y^P\right) \geq I^{\star B}\left(Y^P\right)$ $\forall Y^P$, and, thus, a concordance order:*

i. An increase in the marginal product of investment, $f_I^A(I, H^P) \geq f_I^B(I, H^P)\ \forall\ (H^P, I)$, or an increase in the return to human capital that raises the marginal continuation value of human capital at each H^K;

[31] When f is Cobb-Douglas and W is linear, this produces the standard intergenerational earnings elasticity specification. The timing here follows Loury (1981). In the alternative timing where θ is known at the time of investment (e.g., Cunha, Heckman, and Schennach 2010; Caucutt and Lochner 2020), the pointwise ordering below holds for all (θ, Y^P) pairs. It is also possible to incorporate stochastic wages given human capital; as Lochner and Park (2024) emphasize, intergenerational income mobility will then differ from intergenerational skill mobility. I additionally assume that the income of each child is an increasing function of either human capital or their rank in the human capital distribution, a small abuse of notation. In comparative statics exercises this does not hold the marginal distribution of child incomes constant, but still allows economies to be ranked by the concordance of their copulas. More generally, this sets the model in a general equilibrium context (following Heckman, Lochner, and Taber 1998 or Lee and Wolpin 2006 for example) by allowing wages to adapt to changes in the supply and composition of educated workers without fully specifying general equilibrium forces.

ii. An investment subsidy or policy intervention that increases the marginal return to investment throughout the parental income distribution;

iii. An increase in the discount factor, $\delta^A \geq \delta^B$.

These mechanisms share a common structure: by raising the marginal return to investment, they amplify the role of parental resources in children's human capital.

Illustration. Figure 7 illustrates Lemma 1 in simulated data. As investment returns rise, the joint distribution concentrates along the diagonal (Panels C–D), the conditional expectation function steepens (Panel B), and all mobility measures decline (Panel A).

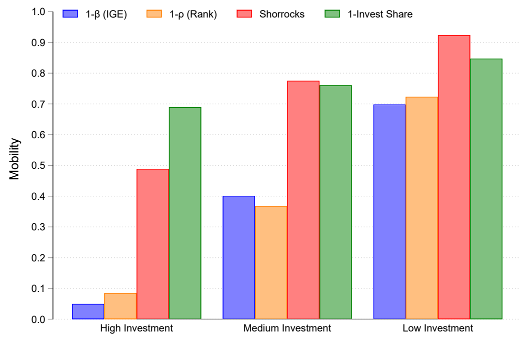
8.2. When Does the Concordance Order Fail?

Not all economically relevant changes expand investment uniformly. School finance equalisation (Jackson, Johnson, and Persico 2016; Biasi 2023) reallocates resources toward poor districts, raising investment at low parental incomes while reducing it at high incomes. The resulting $\Delta I^{\star}(Y^P)$ crosses zero, generating regions where $\Delta C > 0$ and $\Delta C < 0$ which violate the concordance order.[32] Different mobility measures then disagree because they weight these regions differently (Section 5), producing the crossing-copulas pattern detected in Table 2. The concordance order interprets this disagreement; it reflects genuine heterogeneity in how the intergenerational transmission mechanism shifted, not noise or arbitrary choice of statistic.

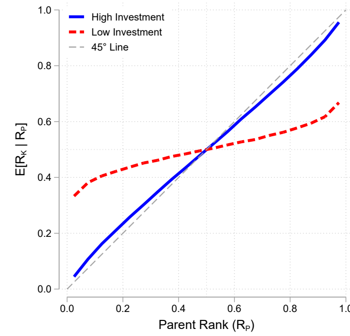
9. Discussion and Implications for Applied Researchers

The core insight is straightforward: when two economies can be ranked by concordance, any mobility measure—ranks, regressions, transition probabilities, or axiomatic distances—agrees on which is more mobile. This answers two open questions in the literature (e.g., Berman 2022; Deutscher and Mazumder 2023). First, when is one measure sufficient? If distributions are concordance-ordered, any single metric serves as a sufficient statistic for all others. Second, which measure should an applied researcher choose? Under concordance, axiomatic arguments for preferring one measure are redundant. When concordance fails, selecting a single measure amounts to choosing a tie-breaking rule, (see also Mogstad et al. 2024). Section 6

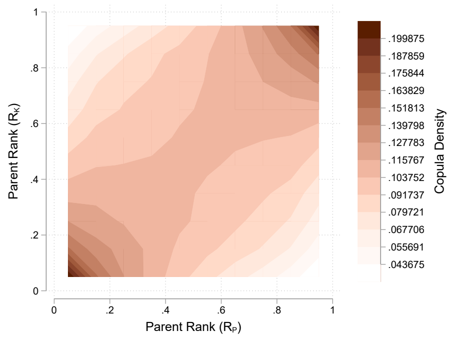
[32] Moreover, reforms will have different impacts on different parts of the parent distribution: precisely the finding in Biasi (2023).



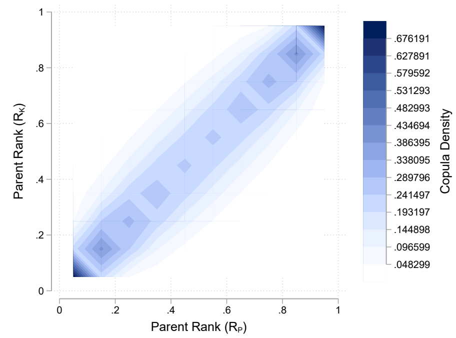
(A) Mobility



(B) Rotation of the CER



(C) Low Investment (High Mobility)



(D) High Investment (Low Mobility)

FIGURE 7. Investment and Mobility in the Becker and Tomes-Loury Model

Notes: Data simulated from the model in Section 8 with $N = 1{,}000{,}000$ parent-child pairs, Cobb-Douglas production, and a log-normal error. Three scenarios vary the returns to investment (α), direct parental human capital effect (γ), and ability shock variance (σ_θ): Low Mobility ($\alpha = 0.50$, $\gamma = 0.45$, $\sigma_\theta = 0.20$), Medium Mobility ($\alpha = 0.35$, $\gamma = 0.25$, $\sigma_\theta = 0.35$), and High Mobility ($\alpha = 0.20$, $\gamma = 0.10$, $\sigma_\theta = 0.50$). Panel (A) reports one minus the IGE ($1 - \beta$), one minus the rank-rank slope ($1 - \rho$), the Shorrocks Trace, and one minus the investment share ($1 - \frac{\alpha\delta}{1+\alpha\delta}$), corresponding to Propositions 1 and 4. Panel (B) reports $\mathbb{E}[R^K \mid R^P]$ for each scenario, illustrating the rotation around the median in Proposition 3. Panels (C) and (D) display the empirical copula density for the Low and High Mobility scenarios on a 10×10 grid of parent-child rank cells; darker regions indicate higher density. Tighter diagonal concentration in Panel (D) reflects greater persistence relative to Panel (C).

provides a structured alternative, by reporting local estimates informed by local violations of the concordance order.

Practical data constraints still guide measure selection, but under concordance this is a matter of convenience rather than substance. Transition probabilities require only ordinal rankings and are robust to measurement error (Corollary 1), making them attractive for local estimates (Chetty et al. 2014a) and settings with noisy incomes. The IGE requires precisely measured log incomes and is sensitive to life-cycle bias

(Haider and Solon 2006) and top-coding. When only proxies are available—education or occupation in historical studies (Ward 2023) or developing countries (Neidhöfer, Serrano, and Gasparini 2018)—Proposition 5 confirms rank-based measures preserve concordance orderings throughout.

The decomposition into dependence and marginal components clarifies a recurring source of empirical confusion. Rank-based measures isolate dependence; the IGE conflates dependence with inequality. The twentieth-century U.S. data confirm this directly: rank-based measures co-move in a wave-like pattern while the IGE diverges as inequality rises after 1950, reflecting changing marginal distributions rather than a strengthening of intergenerational dependence.

The concordance order is not merely statistical. It characterises comparative statics in a standard Becker–Tomes–Loury model: uniform expansions of parental investment increase concordance and lower all mobility measures in tandem, while heterogeneous interventions that strengthen the parent-child link for some families while weakening it for others generate the crossing-copula patterns detected in the data.

Finally, the framework extends beyond intergenerational income mobility to any setting where researchers summarize bivariate dependence structures—intragenerational earnings dynamics, wealth mobility, health transmission, or sorting in marriage and labour markets—provided comparisons operate on relative positions so that dependence can be separated from marginal distributions.

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Online Appendix

A. Measure Agreement for Copula Mobility Measures in Chetty et al. (2014)

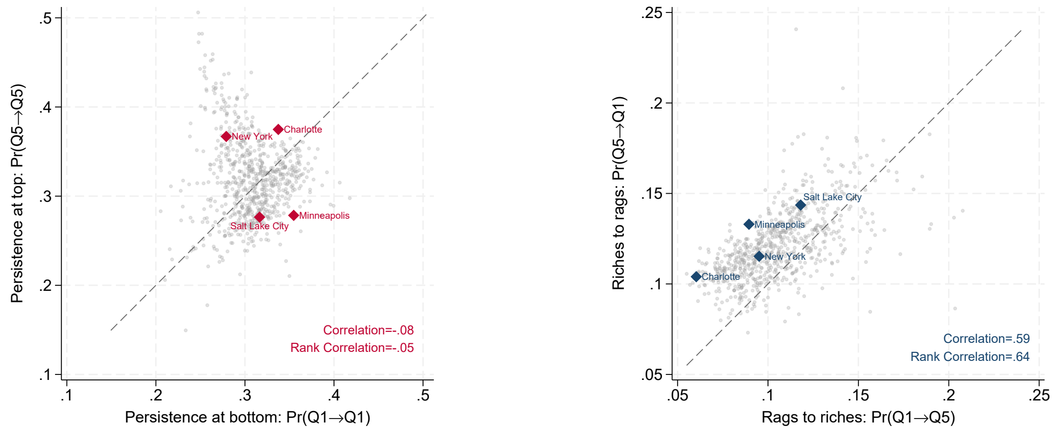
Data and Reconstructing Copulas. I use Commuting Zone Quintile-Quintile Transition Matrices from Chetty et al. (2014) published at https://opportunityinsights.org/data/ which provides transition matrices using quintiles of the national income distribution.

To recover the copula on the joint distribution of within CZ ranks, I remap the observed transition matrix (defined over national quintiles, which need not contain equal shares of the subpopulation) onto a uniform copula grid. This isolates mobility's dependence structure from where the subpopulation sits in the national distribution. The remapping assumes that within each national quintile bin, the subpopulation's ranks are uniformly distributed, i.e., the subpopulation's income density within each national quintile mirrors the national density.

This is a natural approximation given data are only observed at the quintile level. However, the population in the top national quintile may be concentrated at the very top of the CZ specific quintile for high income CZs. Given data limitations there are no other alternatives. Importantly, the results in this Appendix are illustrative and do not change either the main empirical application or the theoretical implications arising from the concordance order.

Measure Consensus. Chetty et al. (2014) document substantial heterogeneity in economic mobility across commuting zones (Table III), with the most and least mobile CZ varying depending on the measure used.

Figure A.1 extends the analysis of measure agreement, showing the joint distribution of measures computed from each CZ's copula. Panel A shows how measures of persistence, the probability of remaining in the same quintile as a parent at the bottom and top of the distribution, covary across commuting zones. These measures of intergenerational persistence are negatively correlated, although it is weak, as many CZs are characterised by much more persistence for high-income parents than for low-income parents. Indeed, the implied correlation is negative. Panel B shows the asymmetric transition probabilities for extreme parent-child reversals.



(A) Regression Estimates (B) Directional Rank-Mobility

FIGURE A.1. Agreement Among Mobility Measures For Chetty et al. (2014)

Notes: See text for data construction details. Panel (A) reports transition probabilities using quintiles of the within-CZ distribution for the probability of remaining in the bottom and top quintiles across generation. Panel (B) reports probilities of transitioning from the bottom quintile to the top quintile and vice versa.

These measures of mobility are more positively correlated, but there are many pairs of CZs where the comparison of who is more mobile depends on which of the measures is chosen. While Salt Lake City is more mobile than Charlotte across any of these measures (as well as the rank-rank slope reported in the paper), whether it is more mobile than New York City or if New York City is more mobile than Minneapolis is sensitive to the choice of measure.

Moreover, a broader definition of measure agreement delivers similar results. Among CZs in the top quartile on either persistence measure (panel A), 91.3% are not in the top quartile based on the other measure. Similarly, among CZs in the top quartile on either tail mobility measure (panel B), 61.1% are not in the top quartile based on the other measure.[33]

B. Discussion and Proofs for Section 2

B.1. Copula Uniqueness

Sklar's Theorem (1959) states that the copula associated with a joint distribution is

[33] Mogstad et al. (2024) discuss the inferential problem for rankings based on estimated measures of mobility. However, the pairwise comparisons above do not have the same concern.

unique if and only if marginal distributions are continuous. When the random variables take on discrete values or are drawn from distributions with mass points, the copula is generically not uniquely determined. Instead, it is only unique on the set of ranks the marginals actually take (Thm 2.3.3, Nelsen 2006). There are infinitely many valid extensions for values outside the observed support of ranks.

Thus, even if mass points are only at the boundary of support (as in the case of censored or truncated income observations), the copula is not unique. The results in this paper do not, unless stated otherwise, rely on continuity or differentiability of the copula. However, they do implicitly assume uniqueness of the copula. Therefore, discreteness represents a problem as it leads to indeterminacy.

Addressing this indeterminacy is referred to as an *extension* of the copula. Unfortunately, extensions are not necessarily guaranteed to preserve the concordance order. Thus, the choice of extension is not without loss. There are three solutions to this problem for a given empirical copula:

(i) Use a checkerboard extension (Genest and Nešlehová 2007) which is piecewise bilinear

(ii) Consider Carley (2002) bounds on copulas

(iii) Approximate the discrete outcomes with a specific parametric copula family.

Solution (iii) is the easiest solution, but also requires the strongest assumptions making it typically unappealing. Solution (ii) would replace inequalities on the copula required for the concordance order with worst-case inequalities comparing upper and lower bounds. This is extremely flexible, but can be limiting when the bounds are not sharp. Finally, solution (i) restricts the set of admissible extensions and selects a bilinear interpolation on the density. This guarantees uniqueness and, as shown by Propositions 11 and 13 in Genest and Nešlehová (2007), the checkerboard extension is the unique extension that preserves the concordance order of the original joint-distributions. Consequently, I assume that discreteness is addressed through checkerboard extension of the copula. While this is a mild assumption, equivalent results can be obtained under either alternative assumption.

B.2. Proofs

LEMMA 1 (Stronger Dependence). *All copula pairs A and B that satisfy Assumption 1 also satisfy the concordance order in Definition 1.*

Proof of Lemma 1. Let $u1 = v1 = 0$, $u_2 = u$ and $v_2 = v$. Then the boundary conditions of the copula give

$$C(0, v) = C(u, 0) = C(0, 0) = 0 \tag{B.1}$$

for all copulas. Thus Assumption 1 is equivalent to

$$\Delta C(u, v) \geq 0, \tag{B.2}$$

which is identical to Definition 1. □

C. Proofs for Section 3

Proof of Proposition 1. The population value of β is given by $cov(\ln y_i^k, \ln y_i^p)/var(\ln y_i^p)$. The denominator is constant and the numerator is increasing relative to the concordance ordering as a direct consequence of Lemma 3 in Lehmann (1966) (see also Lemma 2, Hoeffding 1940). Similarly, the population value of ρ is increasing relative to the concordance ordering from Corollary 3.2. in Tchen (1980). □

Proof of Proposition 2. In the parametric case, the conditional expectation is given by (substituting the closed form value of α):

$$CER = 0.5 + \rho(R - 0.5) \tag{C.1}$$

which is increasing in ρ above the median ($R > 0.5$) and decreasing below ($R < 0.5$). Consequently, given ρ is increasing relative to the concordance ordering (Proposition 1), the stated inequalities follow. This does not rely on Assumption 1.

Define

$$D(v) = \int_0^1 u\, \partial_v C^A(u, v)\, du - \int_0^1 u\, \partial_v C^B(u, v)\, du.$$

Then there exists a unique $v^* \in (0, 1)$ with $D(v^*) = 0$; equivalently, $\mathbb{E}^A[\, U \mid V = v\,]$ and $\mathbb{E}^B[\, U \mid V = v\,]$ cross exactly once.

To establish this we impose an additional regularity assumption: $D(v)$ is single crossing on $[0, 1]$. A stronger sufficient condition for this regularity assumption is to require the family of conditional densities satisfy a monotone-likelihood-ratio or TP_2 properties in models that admit densities or for many parametric copula family. This guarantees uniqueness of the root.

First, consider the left tail of v. We establish that $D(v) > 0$ in the neighborhood of

$v = 0$. Fix $\varepsilon \in (0, 1)$. Using $C^A(u, 0) = C^B(u, 0) = 0$ and $C^A(u, \varepsilon) \geq C^B(u, \varepsilon)$ for every u gives

$$\int_0^{\varepsilon} D(v)\, dv = \int_0^1 u\left[C_1(u, \varepsilon) - C_2(u, \varepsilon)\right] du > 0 \quad (\varepsilon > 0).$$

$D(0) = 0$ and D is continuous, thus some $v_0 \in (0, \varepsilon)$ satisfies $D(v_0) > 0$. Equivalently, in the neighborhood of $v = 1$ $u - C^A(u, 1-\varepsilon) \leq u - C^B(u, 1-\varepsilon)$, hence

$$\int_{1-\varepsilon}^{1} D(v)\, dv < 0.$$

Thus, there exists $v_1 \in (1-\varepsilon, 1)$ with $D(v_1) < 0$. D is continuous, positive at v_0, and negative at v_1; by the Intermediate Value Theorem it has a root $v^* \in (v_0, v_1)$. Under the regularity assumption, a zero can occur *at most once*. Hence v^* is the unique crossing. □

Proof of Proposition 3. The transition probabilities can be rewritten using the conditional CDF. For the positive diagonal case we have

$$TP\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right] = \frac{Pr(R^K \leq \tau^k \cap R^P \leq \tau^p)}{Pr(R^P \leq \tau^p)} = \frac{C(\tau^k, \tau^p)}{\tau^p}, \tag{C.2}$$

and

$$TP\left[R^K > \tau^k \mid R^P > \tau^p\right] = \frac{Pr(R^K > \tau^k \cap R^P > \tau^p)}{Pr(R^P > \tau^p)} = \frac{1 - \tau^p - \tau^k + C(\tau^k, \tau^p)}{1 - \tau^p}, \tag{C.3}$$

with the inequality in the proposition satisfied iff

$$C^A(\tau^k, \tau^p) \geq C^B(\tau^k, \tau^p)\ \forall \tau^k, \tau^p \in [0, 1]\,, \tag{C.4}$$

which is identical to the definition of increased concordance.

For the off-diagonal case we have

$$TP\left[R^K > \tau^k \mid R^P \leq \tau^p\right] = 1 - TP\left[R^K \leq \tau^k \mid R^P \leq \tau^p\right], \tag{C.5}$$

and

$$TP\left[R^K \leq \tau^k \mid R^P > \tau^p\right] = 1 - TP\left[R^K > \tau^k \mid R^P > \tau^p\right], \tag{C.6}$$

with the stated inequality following directly from the results established in the positive diagonal case.

Turning to directional rank mobility. Fix $\tau \in (0, 1]$ and an s such that $1 - \tau \geq s \geq 0$.

First, note that we can express the following conditional density through Bayes' rule

$$Pr(R^K - R^P > s \mid R^P \leq \tau) = \frac{Pr(R^K - R^P > s \cap R^P \leq \tau)}{Pr(R^P \leq \tau)} = \frac{Pr(R^K > R^P + s \cap R^P \leq \tau)}{Pr(R^P \leq \tau)}, \quad \text{(C.7)}$$

where the numerator is the probability that both conditions hold. This can be rewritten as

$$Pr(R^K > R^P + s \cap R^P \leq \tau) = \int_0^\tau Pr(R^K > t + s | R^P = t)\, dt, \quad \text{(C.8)}$$

which uses the fact that ranks are uniformly distributed and bounded between 0 and 1. The conditional density is well-defined (Theorem 2.2.7., Nelsen 2006).

As $Pr(R^P \leq \tau) = \tau$ and is constant across copulas, the stated inequality is identical to establishing the inequality pointwise on the integrand in equation (C.8).

Let

$$\begin{aligned} g_h(x) \equiv Pr(R^K > x + s \mid x < R^P \leq x + h) &= 1 - Pr(R^K \leq x + s \mid x < R^P \leq x + h) \\ &= 1 - \frac{Pr(R^K \leq x + s\, , x < R^P \leq x + h)}{h}, \end{aligned}$$

where the alternative definition is a direct application of Bayes' rule. The integrand in equation (C.8) can then be expressed as

$$Pr(R^K > t + s|R^P = t) = \lim_{h \downarrow 0} g_h(t), \quad \text{(C.9)}$$

where the function $g_h(\cdot)$ is continuous in h by uniform continuity of the copula (corollary 2.2.6., Nelsen 2006). Note that $C(u, v + h) - C(u, v) = Pr(R^K \leq u\, , v < R^P \leq v + h)$. We can then express $g_h(x)$ in terms of copulas as

$$g_h(x) = 1 - \frac{C(x + s, x + h) - C(x + s, x)}{h} = 1 - \frac{Pr(R^K \leq x + s\, , X < R^P \leq x + h)}{h}. \quad \text{(C.10)}$$

Assumption 1 guarantees $\Delta C(u, v + h) \geq \Delta C(u, v)$ and

$$C^A(u, v + h) - C^A(u, v) \geq C^B(u, v + h) - C^B(u, v) \quad \text{(C.11)}$$

$$\longrightarrow \frac{C^A(u, v + h) - C^A(u, v)}{h} \geq \frac{C^B(u, v + h) - C^B(u, v)}{h}. \quad \text{(C.12)}$$

Substituting (C.12) into (C.10), establishes

$$g^A(x) \leq g^B(x) \longrightarrow \lim_{h\downarrow 0} g^A(x) \leq \lim_{h\downarrow 0} g^B(x) \longrightarrow \int_0^\tau \lim_{h\downarrow 0} g^A(t)\,dt \leq \int_0^\tau \lim_{h\downarrow 0} g^B(t)\,dt, \quad \text{(C.13)}$$

thus $Pr^A(R^K > R^P + s \cap R^P \leq \tau) \leq Pr^B(R^K > R^P + s \cap R^P \leq \tau)$. □

Proof of Proposition 4. Fields and Ok Measure. The aggregate income movement per capita is equivalent to

$$\int\int \psi\left(Y^K, Y^P\right) f\left(Y^K, Y^P\right) dY^K dY^P \quad \text{where } \psi(x, y) = |x - y|, \tag{C.14}$$

where the function $\psi(x, y)$ is a convex function of the difference. The result then follows directly as an application of Theorem 9.A.18 in Shaked and Shanthikumar (2007) or Corollary 2.3 (and example 2) in Tchen (1980). Expected absolute deviations across generations are decreasing in the concordance order. The proof in the log-case (Fields and Ok 1999b) is identical. The Fields and Ok (1996, 1999b) measures can be decomposed into structural mobility (an aggregate growth term),

$$\int |Y^K| f^k(Y^K) dY^K - |Y^P| f^P\left(Y^P\right) dY^P, \tag{C.15}$$

which is identical under C^A and C^B with constant marginals as well as a transfer or exchange term

$$2 \times \int\int_{(Y^K, Y^P)\in S} |Y^K - Y^P|\, f(Y^K, Y^P)\, dY^K\, dY^P, \tag{C.16}$$

where the set S selects individuals who are winners or losers. These are dynasties that experience reversals relative to the direction of aggregate income change:

$$S = \begin{cases} \{(Y^K, Y^P) : Y^K < Y^P\}, & \text{if} \quad \overline{Y}^K > \overline{Y}^P \\ \{(Y^K, Y^P) : Y^K > Y^P\}, & \text{if} \quad \overline{Y}^K < \overline{Y}^P \end{cases}. \tag{C.17}$$

It follows that this measure of exchange mobility is decreasing in the supermodular order and thus the concordance order with common marginals.

Cowell and Flachaire Measure The aggregate mobility measure is

$$\int\int \psi_\alpha\left(Y^K, Y^P\right) f\left(Y^K, Y^P\right) dY^K dY^P, \tag{C.18}$$

where the function $\psi_\alpha(x, y) = 1/\alpha(\alpha-1)\,(x/\overline{x})^\alpha\,(y/\overline{y})^{1-\alpha}$ has the following cross-partial derivative:

$$\frac{\psi_\alpha(x, y)}{\partial x \partial y} = -\frac{1}{\overline{x}^\alpha \overline{y}^{1-\alpha}} x^{\alpha-1} y^{-\alpha} \leq 0\,. \tag{C.19}$$

Thus the function is submodular. The result then follows directly as concordance is equivalent to the supermodular order when marginals are constant (Tchen 1980).

Shorrocks Measure Note that the elements stochastic matrix Ω are given by

$$\Omega_{i,j} = Pr\left(\frac{j-1}{q} < R^K \leq \frac{j}{q} \mid \frac{i-1}{q} < R^P \leq \frac{i}{q}\right) \tag{C.20}$$

$$= \frac{1}{q}\left(C\left(\frac{i}{q}, \frac{j}{q}\right) - C\left(\frac{i-1}{q}, \frac{j}{q}\right) - C\left(\frac{i}{q}, \frac{j-1}{q}\right) + C\left(\frac{i-1}{q}, \frac{j-1}{q}\right)\right), \tag{C.21}$$

and the trace is

$$trace(\Omega) = \sum_{i=1}^{q} \Omega_{i,i}. \tag{C.22}$$

It follows that differences in the Shorrocks measure satisfy

$$\begin{aligned}
M^B - M^A &\propto trace\left(\Omega^A\right) - trace\left(\Omega^B\right) \\
&= \frac{1}{q}\sum_{i=1}^{q}\left(C^A\left(\tfrac{i}{q}, \tfrac{i}{q}\right) - C^A\left(\tfrac{i-1}{q}, \tfrac{i}{q}\right) - C^A\left(\tfrac{i}{q}, \tfrac{i-1}{q}\right) + C^A\left(\tfrac{i-1}{q}, \tfrac{i-1}{q}\right)\right) \\
&\quad - \frac{1}{q}\sum_{i=1}^{q}\left(C^B\left(\tfrac{i}{q}, \tfrac{i}{q}\right) - C^B\left(\tfrac{i-1}{q}, \tfrac{i}{q}\right) - C^B\left(\tfrac{i}{q}, \tfrac{i-1}{q}\right) + C^B\left(\tfrac{i-1}{q}, \tfrac{i-1}{q}\right)\right) \\
&= \frac{1}{q}\sum_{i=1}^{q}\left(\Delta C\left(\tfrac{i}{q}, \tfrac{i}{q}\right) - \Delta C\left(\tfrac{i-1}{q}, \tfrac{i}{q}\right) - \Delta C\left(\tfrac{i}{q}, \tfrac{i-1}{q}\right) + \Delta C\left(\tfrac{i-1}{q}, \tfrac{i-1}{q}\right)\right) \geq 0,
\end{aligned}$$

where the final inequality follows directly from Assumption 1. The proof extends immediately to transition matrices with arbitrary cut-points. □

Proof of Proposition 5. This is immediate as an application of the closure properties of the concordance order. See Theorem 9.A.1 Shaked and Shanthikumar (2007). □

Proof of Corollary 1. The result follows immediately from the closure properties of the concordance order (See Theorem 9.A.1 Shaked and Shanthikumar 2007) □

D. Additional Discussion Section 4: Illustration of the Decomposition

The log-normal case makes the decomposition in equation (5) explicit. Suppose parent and child incomes are jointly log-normal:

$$\begin{pmatrix} \ln Y^K \\ \ln Y^P \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_K \\ \mu_P \end{pmatrix}, \begin{pmatrix} \sigma_K^2 & \rho\sigma_K\sigma_P \\ \rho\sigma_K\sigma_P & \sigma_P^2 \end{pmatrix}\right].$$

The copula is Gaussian with parameter ρ with marginals determined by (μ_K, σ_K) and (μ_P, σ_P). For the rank-rank correlation, $\mathrm{Corr}(R^K, R^P) = \frac{6}{\pi}\arcsin(\rho/2) \approx \rho$ (Kruskal 1958), so the marginal component in (5) is exactly zero—changes in σ_K or σ_P have no effect. For the IGE, $\beta = \rho \cdot \sigma_K/\sigma_P$, and the decomposition yields:

$$\Delta\beta = \underbrace{(\rho_B - \rho_A) \cdot \frac{\sigma_K^B}{\sigma_P^B}}_{\text{Copula component}} + \underbrace{\rho_A \cdot \left(\frac{\sigma_K^B}{\sigma_P^B} - \frac{\sigma_K^A}{\sigma_P^A}\right)}_{\text{Marginal component}}.$$

The copula component is ordered by concordance: higher ρ implies higher β, holding marginals fixed. But the marginal component can offset or even reverse this ordering. An economy with lower concordance ($\rho_B < \rho_A$) can exhibit a *lower* IGE if the ratio of child-to-parent dispersion rises sufficiently ($\sigma_K^B/\sigma_P^B \gg \sigma_K^A/\sigma_P^A$).

E. Proofs for Section 5

Proof of Proposition 6. Since $C^A \succeq C^\dagger$ and M is assumed monotone in the concordance order we have $M(C^A) \leq M(C^\dagger)$. By uniform continuity of M we have

$$|M(C^\dagger) - M(C^B)| \leq \omega_M(\|C^\dagger - C^B\|_\infty) \leq \omega_M(\epsilon). \tag{E.1}$$

Therefore $M(C^A) \leq M(C^\dagger) \leq M(C^B) + \omega_M(\epsilon)$ which proves the first result. If M is Lipschitz continuous with constant L_M, then $\omega_M(\epsilon) \leq L_M\epsilon$ which proves the second result. □

F. Additional Results for Section 6: Robust Measure Choice Under Approximate Concordance

Proposition 6 shows concordance results are not a knife-edge property: if two copulas are within ϵ of satisfying concordance (in sup-norm), measures are approximately ordered, with error bounded by ϵ times a measure-specific constant. This matters empirically because real-world copulas may nearly—but not exactly—satisfy concordance due to sampling variation or genuine but minor departures.

The bound L_M varies across measures and determines their sensitivity to concordance violations. Measures that average dependence broadly—such as the rank-rank slope, with $L_\rho = 12$—have smaller Lipschitz constants and degrade smoothly. Measures that concentrate on narrow regions, such as transition probabilities conditioning on rare events, can have larger constants and are more sensitive. This explains why rank-based measures exhibit tighter co-movement in empirical work (Deutscher and Mazumder 2023): their smaller Lipschitz constants make them less sensitive to the minor concordance violations inevitably present in real data. Approximate concordance expands empirical relevance by allowing researchers to test whether copulas satisfy concordance within estimation error. This accommodates realistic settings where economies exhibit small departures due to institutional details or demographics while mobility measures still move together. I now turn to discussing measure selection motivated by robustness to approximate concordance.

A similar approach characterize the sensitivity of measures to these deviations. Recall from equation (8) that any twice-differentiable measure M satisfies[34]

$$M_M^A - M_M^B = \int_0^1 \int_0^1 W_M(R^K, R^P) \left[C^A(R^K, R^P) - C^B(R^K, R^P)\right] dR^K \, dR^P, \tag{F.1}$$

where W_M is the measure specific weighting-kernel with Lipschitz constant:

$$L_M = |W_M|_1 \equiv \int_0^1 \int_0^1 |W_M(u, v)| du \, dv \,. \tag{F.2}$$

COROLLARY 1 (Optimal Robust Measure). *If the true copula C satisfies $||C - C^\dagger||_\infty \leq \epsilon$, then $M(C) \in [M(C^\dagger) - L_M\epsilon, M(C^\dagger) + L_M\epsilon]$. Among concordance-monotone measures, an ϵ-optimal choice of measure minimizes L_M.*

[34] Analogous results apply to discrete cases, but the differentiable case simplifies exposition.

Measure (Table 1)	L_M **for** $\|C_A - C_B\|_\infty$	**Conditions / notes**
IGE (β)	$L_\beta = \dfrac{(\overline{x}^K - \underline{x}^K)(\overline{x}^P - \underline{x}^P)}{\mathrm{Var}(X^P)}$	Fixed marginals; $X^s = \ln Y^s$; bounded log-income support $X^s \in [\underline{x}^s, \overline{x}^s]$. Unbounded support requires additional tail restrictions.
Rank–rank correlation (ρ)	$L_\rho = 12$	Uniform rank marginals (Spearman/Hoeffding representation).
Parametric CER	$L_{\mathrm{CER}(r)} = 12\left\|r - \frac{1}{2}\right\|$	Parametric-only: $\mathrm{CER}_r = \frac{1}{2} + \rho(r - \frac{1}{2})$.
TP	$L = \dfrac{1}{\tau_p}$ for $R_P \leq \tau_p$; $L = \dfrac{1}{1-\tau_p}$ for $R_P > \tau_p$	Requires $\tau_p \in (0,1)$. Direct evaluation bound from copula at (τ_k, τ_p).
Shorrocks	$L_{\mathrm{Sh}} \leq \dfrac{4q\,\kappa_\Omega}{q-1}$	Discrete binning (q groups). The factor κ_Ω collects the normalization used to map copula cell masses into the transition matrix Ω.
Cowell & Flachaire (α)	Levels: $\dfrac{1}{\bar{Y}_K^\alpha \bar{Y}_P^{1-\alpha}}\left(\int y_K^{\alpha-1} f_K(y_K)^2\, dy_K\right) \times \left(\int y_P^{-\alpha} f_P(y_P)^2\, dy_P\right)$ Ranks: $\dfrac{2}{\alpha(1-\alpha)}$, $\alpha \in (0,1)$	Levels: finiteness of the displayed weighted L^2 integrals. Ranks: uniform marginals.

TABLE F.1. Simplified summary of Lipschitz constants used (or not available) for the robust selection rule under the copula sup-norm.

Interpretation. Measures concentrating weight in specific regions (e.g., the intergenerational elasticity or local conditional mobility), have larger L_M and greater sensitivity to concordance violations. As do those measures expressed in larger units. Measures spreading weight uniformly (e.g., Shorrocks index, coarse transitions or rank-rank slopes)—have smaller L_M and are robust to approximate concordance. This explains why rank-based measures exhibit tighter co-movement in empirical work (Deutscher and Mazumder 2023, Table 3).

F.1. Lipschitz Constants for Robust Measure Choice

Throughout, let C_A, C_B be copulas on $[0,1]^2$ with the metric $\|C_A - C_B\|_\infty \equiv \sup_{(u,v)\in[0,1]^2} |C_A(u,v) - C_B(u,v)|$. For any functional M that admits the representation in (F.3),

$$M(C_A) - M(C_B) = \int_0^1 \int_0^1 W_M(u,v)\left(C_A(u,v) - C_B(u,v)\right) du\, dv, \qquad L_M = |W_M|_1, \tag{F.3}$$

the Lipschitz bound follows immediately:

$$|M(C_A) - M(C_B)| \leq |W_M|_1 \, \|C_A - C_B\|_\infty. \tag{F.4}$$

To maintain simple exposition I focus on cases where the measure is twice-differentiable or can be bounded directly.

F.1.1. IGE (β)

Let $X^K = \ln Y^K$ and $X^P = \ln Y^P$, with fixed marginal distributions F_{X^K} and F_{X^P} and corresponding densities f_{X^K} and f_{X^P}. The intergenerational elasticity is

$$\beta(C) = \frac{Cov(X^K, X^P)}{Var(X^P)},$$

where $Var(X^P)$ is constant under fixed marginals. By Hoeffding's identity and Sklar's theorem,

$$\begin{aligned} Cov(X^K, X^P) &= \int\int \left(F_{X^K,X^P}(x^K, x^P) - F_{X^K}(x^K)F_{X^P}(x^P)\right) dx^K\, dx^P \\ &= \int_0^1 \int_0^1 \frac{C(u,v) - uv}{f_{X^K}\left(F_{X^K}^{-1}(u)\right)\, f_{X^P}\left(F_{X^P}^{-1}(v)\right)}\, du\, dv. \end{aligned} \tag{F.5}$$

Hence, in the notation of equation (8), the weighting kernel for the IGE is

$$W_\beta(u,v) = \frac{1}{Var(X^P)} \frac{1}{f_{X^K}\left(F_{X^K}^{-1}(u)\right)\, f_{X^P}\left(F_{X^P}^{-1}(v)\right)}. \tag{F.6}$$

It follows that, on bounded support for log-incomes,

$$X^K \in [\underline{x}^K, \overline{x}^K], \qquad X^P \in [\underline{x}^P, \overline{x}^P],$$

the IGE is Lipschitz in the sup-norm with constant

$$\begin{aligned}
L_\beta &= |W_\beta|_1 \\
&= \frac{1}{Var(X^P)} \int_0^1 \int_0^1 \frac{1}{f_{X^K}\left(F_{X^K}^{-1}(u)\right) \, f_{X^P}\left(F_{X^P}^{-1}(v)\right)} \, du \, dv \\
&= \frac{1}{Var(X^P)} \left(\int_{\underline{x}^K}^{\overline{x}^K} dx^K \right) \left(\int_{\underline{x}^P}^{\overline{x}^P} dx^P \right) \\
&= \frac{(\overline{x}^K - \underline{x}^K)(\overline{x}^P - \underline{x}^P)}{Var(X^P)}.
\end{aligned} \tag{F.7}$$

Therefore,

$$|\beta(C^A) - \beta(C^B)| \leq L_\beta \, \|C^A - C^B\|_\infty.$$

The same argument applies on any trimmed support of log-incomes. When log-incomes are unbounded, a finite global Lipschitz constant need not exist without additional tail restrictions.

F.1.2. **Rank–rank slope** (ρ)

With uniform rank marginals, the rank–rank slope is $\rho = \mathrm{Cov}(U, V)/\mathrm{Var}(V)$, and $\mathrm{Var}(V) = 1/12$. Hoeffding's identity gives $\mathrm{Cov}(U, V) = \int_0^1\int_0^1 (C(u, v) - uv) \, du \, dv$, so

$$\rho(C) = 12 \int_0^1 \int_0^1 \left(C(u, v) - uv\right) du \, dv.$$

Thus

$$\rho(C_A) - \rho(C_B) = 12 \int_0^1 \int_0^1 \left(C_A(u, v) - C_B(u, v)\right) du \, dv,$$

and

$$|\rho(C_A) - \rho(C_B)| \leq 12 \iint |C_A - C_B| \leq 12 \, \|C_A - C_B\|_\infty.$$

Hence

$$L_\rho = 12.$$

F.1.3. **CER**

In the parametric case the conditional expected rank at parent rank r is

$$\mathrm{CER}_r(C) = \tfrac{1}{2} + \rho(C) \, (r - \tfrac{1}{2}),$$

so Lipschitz continuity follows by composition with ρ:

$$|\mathrm{CER}_r(C_A) - \mathrm{CER}_r(C_B)| = |r - \tfrac{1}{2}|\,|\rho(C_A) - \rho(C_B)| \leq 12|r - \tfrac{1}{2}|\,\|C_A - C_B\|_\infty.$$

Therefore

$$L_{\mathrm{CER}(r)} = 12|r - \tfrac{1}{2}|.$$

F.1.4. TP (transition probabilities)

Appendix C rewrites diagonal transition probabilities as evaluations of C:

$$\mathrm{TP}\big[R_K \leq \tau_k \,\big|\, R_P \leq \tau_p\big] = \frac{C(\tau_k, \tau_p)}{\tau_p}, \qquad \tau_p \in (0,1).$$

Hence

$$\Big|\mathrm{TP}_A - \mathrm{TP}_B\Big| = \frac{|C_A(\tau_k, \tau_p) - C_B(\tau_k, \tau_p)|}{\tau_p} \leq \frac{1}{\tau_p}\,\|C_A - C_B\|_\infty,$$

so

$$L_{\mathrm{TP}\leq} = 1/\tau_p.$$

Similarly, for upper-tail conditioning

$$\mathrm{TP}\big[R_K > \tau_k \,\big|\, R_P > \tau_p\big] = \frac{1 - \tau_p - \tau_k + C(\tau_k, \tau_p)}{1 - \tau_p},$$

and the same argument yields

$$L_{\mathrm{TP}>} = 1/(1 - \tau_p).$$

F.1.5. Shorrocks (trace measure)

Ffor q bins the discretized transition matrix is

$$Q_{ij}(C) = \kappa_Q\Big(C(\tfrac{i}{q}, \tfrac{j}{q}) - C(\tfrac{i-1}{q}, \tfrac{j}{q}) - C(\tfrac{i}{q}, \tfrac{j-1}{q}) + C(\tfrac{i-1}{q}, \tfrac{j-1}{q})\Big),$$

and $\mathrm{trace}(Q) = \sum_{i=1}^{q} Q_{ii}$. Let $\Delta C = C_A - C_B$. Then each diagonal increment is a signed sum of four evaluations of ΔC, so

$$|Q_{ii}(C_A) - Q_{ii}(C_B)| \leq 4\kappa_Q\,\|\Delta C\|_\infty.$$

Summing over i yields

$$|\mathrm{trace}(Q(C_A)) - \mathrm{trace}(Q(C_B))| \leq 4q\kappa_Q \, \|C_A - C_B\|_\infty.$$

For Shorrocks' trace index $\mathrm{Sh}(Q) = (q - \mathrm{trace}(Q))/(q-1)$, it follows that

$$|\mathrm{Sh}(C_A) - \mathrm{Sh}(C_B)| \leq \frac{4q\kappa_Q}{q-1} \, \|C_A - C_B\|_\infty, \qquad L_{\mathrm{Sh}} \leq \frac{4q\kappa_Q}{q-1}.$$

(Here κ_Q records the bayes rule normalization used in the appendix to convert cell masses to Q.)

F.1.6. Cowell & Flachaire (parameter α)

Using equation (C.19), which yields

$$W_{\mathrm{CF}}(u,v) = \left|\psi_{\alpha,12}(F_K^{-1}(u), F_P^{-1}(v))\right| \, f_K(F_K^{-1}(u)) \, f_P(F_P^{-1}(v)).$$

Thus $L_{\mathrm{CF}} = |W_{\mathrm{CF}}|_1$ and, by the same change-of-variables step used above,

$$L_{\mathrm{CF}} = \frac{1}{\bar{Y}_K^{\alpha} \bar{Y}_P^{1-\alpha}} \left(\int y_K^{\alpha-1} f_K(y_K)^2 \, dy_K \right) \left(\int y_P^{-\alpha} f_P(y_P)^2 \, dy_P \right),$$

and if the measure is evaluated on ranks (uniform marginals), the same calculation simplifies to the closed form reported in Table F.1: $L_{\mathrm{CF}} = 2/(\alpha(1-\alpha))$ for $\alpha \in (0,1)$.

G. Additional Results for Section 6: Connection to Models of Scientific Communication

The reporting protocol in Section 6.2 can be understood through the lens of Andrews and Shapiro (2021), who study when a statistical report is adequate for a heterogeneous audience.

In their framework, an analyst makes a report to an audience after observing data. Audience members differ in their objectives and may therefore act differently following a given report. A report is *communication-admissible* if no alternative report would make all audience members weakly better off and at least one strictly better off.

The mobility setting maps directly into this framework. The analyst is a researcher comparing two economies. The audience consists of readers who care about different mobility measures—one cares about the IGE, another about bottom-quintile transitions, a third about the Shorrocks trace. Formally, each audience member m has a weighting kernel W_m and evaluates the comparison through the integral $\int\int W_m \cdot \Delta C$. The researcher's report is whatever summary statistics they publish.

Single-measure reporting is communication-inadmissible. When concordance holds, any single concordance-monotone measure determines the sign of all others. A report consisting of the global rank-rank slope alone is communication-admissible: every audience member can determine whether their preferred measure ranks A above or below B.

When concordance fails, the global rank-rank slope is communication-inadmissible. The argument follows directly from Corollary 2. There exist audience members—specifically, readers whose preferred measures are transition probabilities at different thresholds—who reach opposite conclusions. The global slope integrates ΔC against a uniform kernel, reflecting one net direction. A reader whose preferred measure loads on a region where ΔC takes the opposite sign is strictly worse off than under a report that disclosed the local information.

This is directly analogous to the deworming example in Andrews and Shapiro (2021): censoring negative treatment effect estimates discards information that some audience member could use. Reporting only the global rank-rank slope when copulas cross discards information about *where* concordance fails—information that some audience member needs.

Local slopes as neutral communication. The rank-rank slope has a constant intensity kernel: $W_\rho(u, v) = 12$ for all (u, v). This makes the local rank-rank slope the most "neutral" local report in the following sense. For any audience member with kernel W_m, the local slope in bin B_j captures the unweighted average of ΔC in that bin:

$$\Delta\rho_j \propto \int_{B_j} \int_0^1 \Delta C(u, v)\, du\, dv. \tag{G.1}$$

An audience member with smooth kernel W_m can approximate their measure's local contribution as

$$\int_{B_j} \int_0^1 W_m(u, v)\, \Delta C(u, v)\, du\, dv \approx \bar{W}_{m,j} \cdot \Delta\rho_j\,, \tag{G.2}$$

where $\bar{W}_{m,j}$ is the average of W_m within bin j. The approximation error depends on the variation of W_m within the bin and vanishes as the partition refines. Because the local slope itself uses uniform weights, this approximation is equally good for all smooth-kernel audience members—no particular reader is systematically favoured or disfavoured by the choice of local statistic.

For audience members with concentrated kernels—readers who care about a specific transition probability—the local slope may be insufficient because it averages over the point they care about. Supplementing with transition probabilities at standard thresholds (step (c) of the protocol) addresses this directly. The combination of local slopes and selected transition probabilities is thus approximately communication-admissible for the class of concordance-monotone measures encompassing both smooth and concentrated intensities.

Minimum partition size. The Lipschitz bounds in Appendix F provide a formal criterion for the coarseness of the reporting partition. For a measure with Lipschitz constant L_M, the error from using a J-bin partition rather than the full ΔC surface is of order L_M/J. A partition is communication-admissible for the class of measures with Lipschitz constant bounded by $\bar{L}$ when $\bar{L}/J$ is small relative to estimation uncertainty. For the standard measures in Table 1, this is typically achieved with a small number of regions.

H. Additional Results for Section 6: Welfare-Consistent Completion of the Concordance Order

An alternative approach to completing the concordance order incorporates welfare judgments. A planner ranks economies based on distributional preferences over parent-child outcomes.

Reinterpreting equation (7), the planner's choice of measure implicitly assigns welfare weights $\phi_M(Y^K, Y^P)$. When $\phi_M(Y^K, Y^P)$ is submodular, the planner has a preference for a decrease in concordance. This completion respects concordance rankings while breaking ties via planner preferences. It accommodates utilitarian welfare with concave utility over dynastic income, opportunity-equality objectives minimising rank correlation, and poverty-focused weights concentrating on low-rank dynasties. This extends results in Atkinson and Bourguignon (1982) to all submodular social welfare functions.

I. Construction of the Replication Dataset

The construction of the data follows Jácome, Kuziemko, and Naidu (2025). I briefly summarize the key features of their approach here and refer the interested reader to Jácome, Kuziemko, and Naidu (2025).

Data sources. I pool the fifteen U.S. surveys identified by Jácome, Kuziemko, and Naidu (2025) that report (i) respondents' current family income, (ii) father's occupation when the respondent was growing up, (iii) respondent race, and (iv) birthplace or childhood region (South vs. non-South). The surveys include American National Election Studies, General Social Survey, the Panel Study of Income Dynamics (specifically 1997 and 2017 waves), Occupational Changes in a Generation 1962 and 1973 surveys, the National Longitudinal Surveys, and others listed in Appendix E of their paper.

I use IPUMS census extracts from 1910 to 2019 (American Community Survey). These extracts differ from the original analysis which uses the full 1940 census available on the NBER server. They may also differ in other years due to differences in the size of the census-extracts selected.

Sample. I retain U.S-born men and women aged 30–50, the window that best approximates permanent income while minimising life-cycle bias . Respondents must have non-missing family income, race, region, and father's occupation. Foreign-born individuals are excluded because parental incomes are imputed from U.S. sources.

Respondent income. Each survey's family-income question is harmonised into 10–12 real-1950-dollar bins; continuous responses are recoded to the bin mid-point. Bottom-coded observations (including 0) are set to 0.75 × the upper boundary of this lowest bin and those in the top-bin are set to 1.25 × lower-bin boundary.

Parental income imputation. Fathers' (and, where available, mothers') occupations are mapped into 28 broad categories (e.g., skilled crafts, farm operators) . Predicted parental family income is the mean household income in matching occupation × race × South cells drawn from historical micro-data: 1901 Cost of Living Survey & 1900 Census (early cohorts), full-count 1940 Census plus the 1936 Expenditure Survey, and 1960–1990 Censuses. For farmers and self-employed, incomes are adjusted following Collins and Wanamaker (2022); non-working fathers receive values imputed from contemporaneous

Census means. Father race is proxied by respondent race; father region by respondent childhood region.

Weighting. Where surveys supply sampling weights, I re-centre them to mean 1. I then adjust all surveys so that each birth-decade cell matches Census race-sex population shares (white men/women, Black men/women); surveys lacking weights receive weight equal to 1 before adjustment.

Cohorts. Decade birth cohorts 1910s–1970s are used. For each respondent (a) log family income and (b) percentile rank in the pooled income distribution are computed; parental income is treated analogously, yielding comparable distributions of income ranks across cohorts.

These steps reproduce the long-run, nationally representative parent–child dataset used in the original study and permit the replication of Figure 1 in their paper (Panel (A), Figure 5 here).

J. A Testing Procedure for the Concordance Order

I evaluate the concordance ordering condition on a finite grid $\mathcal{G} = \{(u_g, v_g)\}_{g=1}^{G}$ with $u_g = j_1/J$ and $v_g = j_2/K$ for $j_1 = 1, \ldots, J-1$ and $j_2 = 1, \ldots, K-1$, giving $G = (J-1)(K-1)$ interior grid points.[35] Define

$$m_g = C_A(u_g, v_g) - C_B(u_g, v_g), \qquad g = 1, \ldots, G. \tag{J.1}$$

The null hypothesis of concordance dominance becomes

$$H_0: \ m_g \geq 0 \ \text{ for all } g = 1, \ldots, G \qquad \text{vs.} \qquad H_1: \ m_g < 0 \ \text{ for some } g. \tag{J.2}$$

This is a standard moment inequality testing problem with G inequality restrictions. I use $J = K = 10$ (giving $G = 81$).

Let $\{(Y^s_{k,i}, Y^s_{p,i})\}_{i=1}^{n_s}$ be independent random samples from population $s \in \{A, B\}$, with

[35]This is a standard approach in the univariate stochastic dominance setting, but it is consistent only against alternatives that violate $C_A \geq C_B$ at the fixed grid point. A finer grid increases power against localized alternatives at the cost of computation. Intuitively, the granularity of the grid should increase as the sample size increases. As the variance of comparisons in the tails can be very large, it also serves to effectively truncate the tails.

the two samples independent of each other. Define the empirical copula for group s as

$$\widehat{C}_s(u, v) \;=\; \frac{1}{n_s} \sum_{i=1}^{n_s} \mathbf{1}\Big\{ \hat{U}^s_{k,i} \leq u, \;\; \hat{U}^s_{p,i} \leq v \Big\}, \tag{J.3}$$

where $\hat{U}^s_{k,i} = R^s_{k,i}/(n_s + 1)$ and $\hat{U}^s_{p,i} = R^s_{p,i}/(n_s + 1)$ are the rescaled within-group ranks of child and parent outcomes. The sample moment is

$$\widehat{m}_g \;=\; \widehat{C}_A(u_g, v_g) - \widehat{C}_B(u_g, v_g). \tag{J.4}$$

The key distributional result is the joint central limit theorem for the empirical copula process:

$$\sqrt{n_s}\big(\widehat{C}_s(\cdot) - C_s(\cdot)\big) \;\to^d\; \mathbb{G}_s(\cdot), \tag{J.5}$$

where $\mathbb{G}_s$ is a tight Gaussian process on $[0, 1]^2$ whose covariance structure depends on C_s and its partial derivatives (Fermanian, Radulovic, and Wegkamp 2004). By independence of the two samples and the continuous mapping theorem, it follows that

$$\big(\sqrt{n_A}\,(\widehat{C}_A - C_A),\; \sqrt{n_B}\,(\widehat{C}_B - C_B)\big) \;\to^d\; (\mathbb{G}_A, \mathbb{G}_B) \tag{J.6}$$

jointly, with $\mathbb{G}_A$ and $\mathbb{G}_B$ independent. Evaluated at the G grid points, this implies joint asymptotic normality of $\widehat{m} = (\widehat{m}_1, \ldots, \widehat{m}_G)'$:

$$\widehat{m} - m \;=\; (\widehat{C}_A - C_A)\big|_{\mathcal{G}} \;-\; (\widehat{C}_B - C_B)\big|_{\mathcal{G}} \;\approx\; N\Big(0,\; \frac{\Sigma_A}{n_A} + \frac{\Sigma_B}{n_B}\Big),$$

where Σ_s denotes the $G \times G$ covariance matrix of the Gaussian limit $\big(\mathbb{G}_s(u_g, v_g)\big)_{g=1}^{G}$.

J.1. Test Statistic

The test statistic is the minimum of the studentized moments:

$$T_n \;=\; \min_{g=1,\ldots,G} \frac{\widehat{m}_g}{\widehat{s}_g}, \tag{J.7}$$

where $\widehat{s}_g$ is the bootstrap standard error of $\widehat{m}_g$ which incorporates dependence introduced by rank estimation in constructing the empirical copula. I use i.i.d. resampling that recomputes ranks within resamples. Under H_0, $m_g \geq 0$ for all g, so T_n tends to be non-negative. Large negative values provide evidence against the null; if all inequalities are strictly slack T_n approaches $+\infty$.

I adopt the generalized moment selection (GMS) procedure of Andrews and Soares (2010) (see also Andrews and Barwick 2012) by using the data to determine which moments are near-binding and which are clearly slack, and constructs critical values that reflect only the near-binding moments. This yields a test that is asymptotically valid and less conservative than other alternatives. Following Andrews and Soares (2010), I set $\kappa_n = \sqrt{2 \log \log n}$ where $n = n_A + n_B$. This sequence diverges slowly, ensuring that truly slack moments are eventually detected while near-binding moments are not prematurely discarded. The resulting recentered bootstrap distribution approximates the null distribution where the near-binding moments are exactly at the boundary $m_g = 0$, while ffectively excludeing the slack moments entirely.

Note, Andrews and Soares (2010) state their results for sample averages of i.i.d. observations, which $\widehat{m}_g$ is not: the empirical copula involves rank estimation, so $\widehat{m}_g$ is not a simple mean. However, their key requirements are (i) the joint CLT for $\sqrt{n}(\widehat{m} - m)$, (ii) consistency of the bootstrap covariance estimator, and (iii) validity of the bootstrap for approximating the distribution of $\sqrt{n}(\widehat{m} - m)$. All three hold: (i) follows from Fermanian, Radulovic, and Wegkamp (2004), and (ii)–(iii) follow from the bootstrap validity results for empirical copula processes in Rémillard and Scaillet (2009).

Relative to the intersection–union min-t procedures used in the concordance-order literature (e.g., Denuit and Scaillet 2004.), I implement the grid-based dominance test as a modern moment-inequality problem and use GMS/bootstrap critical values to reduce conservativeness when many inequalities are slack.

J.2. Two-Sided Assessment

To assess whether the copulas can be ranked, we run the test in both directions: first testing $H_0 : C_A \succeq C_B$, then $H_0 : C_B \succeq C_A$. Four outcomes are possible:

Reject $C_A \geq C_B$?	Reject $C_B \geq C_A$?	Interpretation
No	No	No significant ordering detected
Yes	No	$C_B \succ C_A$ (B more concordant)
No	Yes	$C_A \succ C_B$ (A more concordant)
Yes	Yes	Copulas cross (no concordance ordering)

K. Additional Results for Section 7

Figure K.1 repeats the exercise of Section 7 for two additional axiomatic indices. Panel (A) plots the exchange mobility measure of Fields and Ok; Panel (B) shows the Cowell and Flachaire measure with "relative status" computed using income ranks and levels. Both rise across successive birth cohorts, echoing the upward trends in rank-based measures from Figure 5, even though the concordance order does not, by itself, determine the behaviour of these measures when marginals differ across cohorts.

By contrast, the absolute version of the Fields and Ok index, which incorporates growth in mean income and changes in inequality, tells a different story. It climbs sharply for early-century cohorts but flattens thereafter, much like the reversal in the IGE. Aggregate income growth concentrated in the right tail of the income distribution (see, e.g., Blundell et al. 2018; Piketty, Saez, and Zucman 2018; Guvenen et al. 2022) offsets the gains in relative mobility, leaving absolute mobility roughly constant in the latter half of the century.

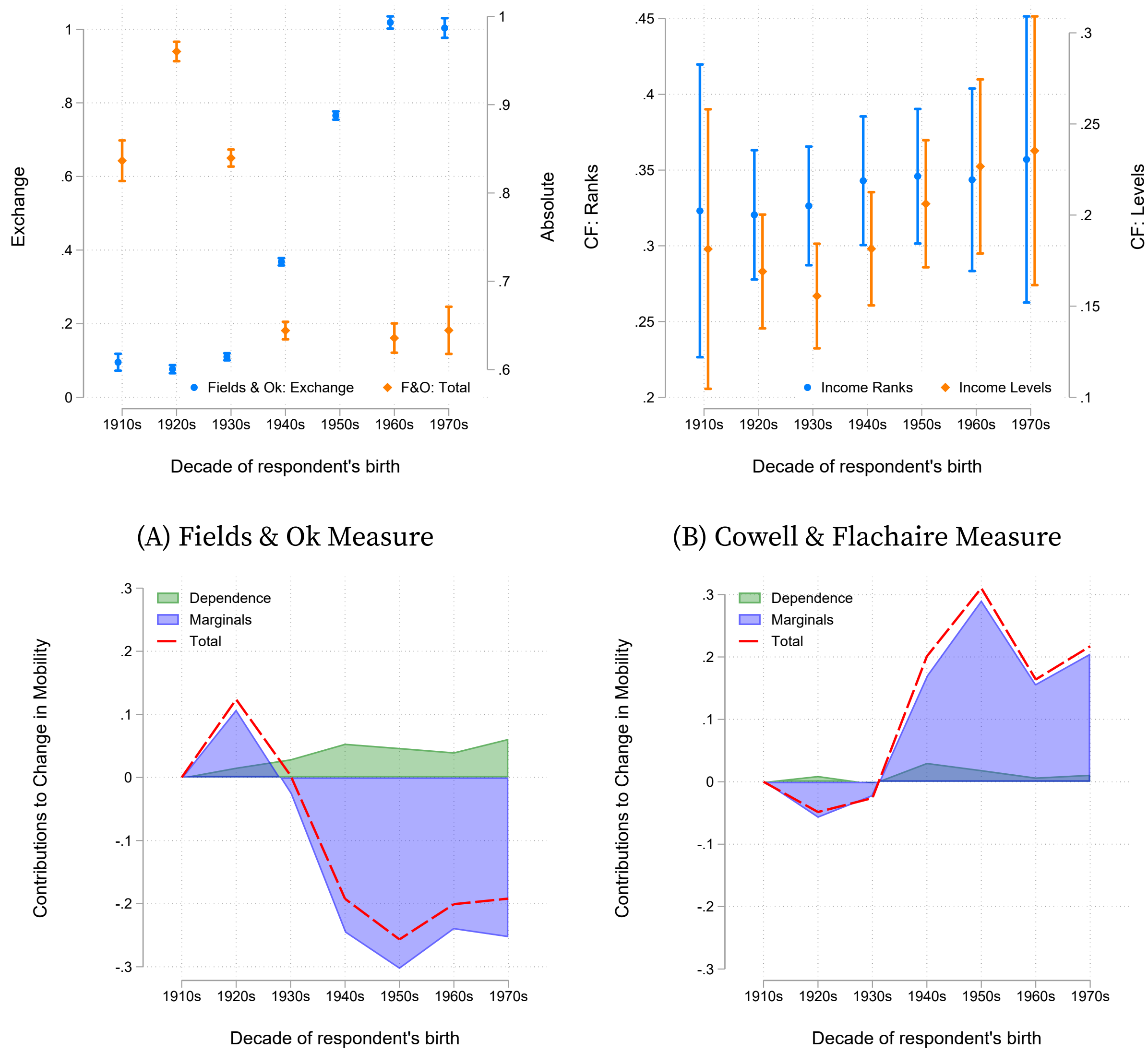
The decomposition in equation (6) allows me to quantify how much of the change in each measure reflects shifts in the dependence structure versus changes in marginal distributions. Figure 6 reports results using Shapley-Owen-Shorrocks averaging over both orderings. This answers why level-based measures diverge from rank-based measures in practice.

Panels (D) and (E) show that, like the measures reported in the main text, Fields and Ok measures similarly decompose into dependence-driven gains and marginal changes. In relative units, the decomposition in equation (6) reveals that these are poorly suited to measuring changes in dependence and, instead, are domianted by changing marginals. In this application, the mobility measures that depend on the level of (log-)income are both highly sensitive to and dominated by the evolution of the income distribution over the 20th century.

L. Proofs for Section 8

Proof of Lemma 1. As $I^{*A}(Y^P) \geq I^{*B}(Y^P)$ for all Y^P and f is increasing in I, we have

$$H^{K,A} \mid Y^P = y \quad \succeq_{\text{FOSD}} \quad H^{K,B} \mid Y^P = y \qquad \forall y.$$



(A) Fields & Ok Measure

(B) Cowell & Flachaire Measure

(C) Fields & Ok (Total)

(D) Fields & Ok (Transition)

FIGURE K.1. Trends in Intergenerational Mobility Measures (cont.)

Notes: See Figure 4 for data construction details. Panels (A) and (B) report estimates of the Fields & Ok and Cowell & Flachaire measures (Proposition 4), respectively. Panels (C) and (D) report Fields and Ok (1996, 1999a) total mobility and exchange (transition) mobility measures, respectively. Each panel decomposes the change in mobility (relative to the 1910s cohort) into a dependence (copula) component and a marginal component following equation (6), with Shapley averaging over both orderings to address path dependence.

Applying a common CDF (e.g. B), the rank variables satisfy

$$F_{R^{K,A}|Y^P}(r \mid y) \;\le\; F_{R^{K,B}|Y^P}(r \mid y) \qquad \forall r \in (0,1),\ \forall y, \tag{L.1}$$

where r is the r^{th} quantile of $H^{K,B}$'s distribution.

Now consider the three permissable cases for $W(H^K)$ outline in the main-text:

Case 1: Rank-dependent wages ($F_W^A(w) = F_W^B(w)\ \forall w$). This implies the marginal distributions of wages are held fixed in the two economies. Integrating inequality (L.1) gives the bivariate joint density as Y^P has the same marginal density f_{Y^P} in both economies. Hence

$$(R^K, Y^P)^A \succeq (R^K, Y^P)^B.$$

Which is equivalent to the concordance ordering as the copula is preserved under monotone transformations.

Case 2: Common monotone-increasing function of human capital ($W^A(H^K) = W^B(H^K) = W(H^K)$). In this case, an immediate consequence of (L) is that child incomes in economy A first order stochastic dominate those in economy B.

Thus, by decomposing the supermodular order into marginals and copulas (Thm 2. Meyer and Strulovici 2013) we have an increase in concordance.

Case 3: Increased returns to human capital ($W^A(H^K) \geq W^B(H^K)\ \forall H^K$). Here analogous reasoning applies.

□

Proof of Corollary 3. For any Y^P, let $V(I;\lambda) = U\left(C^P\right) + \delta\mathbb{E}_\theta\left[U\left(W_\lambda(H^K)\right)\right]$ denote the value of the objective function for choice I and parameter λ controls the counterfactual. The proof follows from Topkis theorem under increasing differences. If the cross-partial (or discrete analogue) satisfies

$$\frac{\partial^2 V}{\partial I \partial \lambda} = \frac{\partial}{\partial \lambda}\left[\delta \frac{\partial f(I, H^P)}{\partial I}\mathbb{E}_\theta\left[U'\left(W_\lambda(H^K)\right) W'_\lambda(H^K)\right]\right] \geq 0, \tag{L.2}$$

then investment shifts outwards as the policy parameter λ increases. Under mild regularity conditions on general equilibrium effects,[36] it is easy to verify that the stated counterfactuals satisfy increasing differences whenever they raise the marginal return

[36] A sufficient condition is that $U'(W^A(H^K))W^{A\prime}(H^K) \geq U'(W^B(H^K))W^{B\prime}(H^K)$ for all H^K. Policy counterfactuals in quantitative models (e.g., Abbott et al. 2019; Lee and Seshadri 2019) typically satisfy such conditions. See Topkis (1998) or Milgrom and Shannon (1994) for general treatments of increasing differences. For multiplicative wage shifts, $W_\lambda(H) = g(\lambda)W_0(H)$ with $g'(\lambda) > 0$, this holds under log utility.

to investment pointwise, either through f_I, through the marginal continuation value $U'(W_\lambda(H^K))W'_\lambda(H^K)$, or through δ.

$\square$

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