A Note on Warm-Glow Bequests and Descendant Risk

Rory M^c^Gee

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Abstract

Reduced-form bequest motives are routinely assumed to exhibit decreasing absolute risk aversion (DARA). I show this is always consistent with an underlying altruistic primitive: when descendant risk is unrealised at the bequest choice date, DARA of the warm-glow preference is inherited from the descendant's continuation value under both multiplicative and additive risk, without parametric restriction. Yet the two canonical specifications differ in what is preserved beyond DARA. Multiplicative risk yields a shifter-weight decomposition and preserves CRRA form; additive income risk—the relevant case for dynastic precautionary savings—preserves DARA but not functional form.

JEL: D64, D81, E21. **Keywords:** bequests, risk aversion, precautionary saving.

1 Introduction

Warm-glow bequest motives are a standard device in quantitative macroeconomics and structural models of life-cycle consumption savings decisions. A parent is assumed to value bequests directly, and the descendant's problem need not be modelled in detail. Following Abel and Warshawsky (1988), who showed that altruism under certainty can be reduced to this form, a large applied literature has adopted warm-glow preferences with decreasing absolute risk aversion—often CRRA or a shifted power function—and calibrated them to wealth and bequest data.[1]

Whether DARA is actually implied by an underlying altruistic framework, and what else is implied, depends on the structure of descendant risk. This dependence has received little attention in the literature, despite growing empirical evidence that the dynastic precautionary motive—parents saving to insure children against income risk—is quantitatively important (Boar, 2021).

This paper provides a characterisation. I establish that DARA of the reduced-form bequest motive is inherited from DARA of the descendant's continuation value under both multiplicative and additive risk, for arbitrary non-parametric preferences. The proof relies on a Cauchy–Schwarz inequality applied to the derivative characterisation of DARA and is self-contained. The result

[1] Among many others, De Nardi (2004), Hurd (1989), Kopczuk and Lupton (2007), Ameriks et al. (2011) and De Nardi et al. (2025).

justifies the widespread DARA assumption but is also limited: DARA is a weak restriction, and a natural question is what further discipline is provided by the risk structure of the economy?

On this question the two canonical specifications diverge. Multiplicative descendant risk—arising when bequests earn idiosyncratic returns (e.g. Constantinides and Duffie, 1996)—preserves functional form: under pure CRRA period utility, the warm-glow bequest motive is itself CRRA with the same curvature parameter and a constant risk adjustment independent of the bequest level. This is similar to results under certainty derived by Abel and Warshawsky (1988). Additive income risk—the formulation in models following Bewley (1986), Huggett (1993), and Aiyagari (1994) or Deaton (1989), Zeldes (1989), Carroll (1997)—preserves DARA but not functional form. Thus, the reduced-form preference for bequests differs from the primitive of descendant utility. This delivers a risk adjustment that varies with bequests, and no shifter-weight decomposition obtains. Since additive income risk is the relevant specification for the dynastic precautionary channel documented by Boar (2021) and for the broader class of heterogeneous-agent models in which warm-glow bequests appear, this asymmetry has practical content: in these settings, the researcher's choice of functional form is not disciplined by the underlying altruism.

2 Setup

A parent chooses bequest $b \geq 0$ before the realisation of shocks affecting the descendant's lifetime resources. The descendant's continuation problem is arbitrary; write $V^*(x)$ for expected continuation utility as a function of initial resources x, with $V^{*\prime} > 0$, $V^{*\prime\prime} < 0$, and V^* three times differentiable on its domain. Resources depend on the bequest and on a shock ζ unrealised at the bequest date: $x = h(b, \zeta)$.

Since the parent cannot condition on ζ, the expected descendant payoff is

$$v(b) = \mathbb{E}[V^*(h(b,\zeta))]\,, \tag{1}$$

a deterministic function of b—the warm-glow representation. Under *multiplicative risk*, $x = bZ$ with $Z > 0$ and $\mathbb{E}[Z] = 1$. Under *additive risk*, $x = b + Y$ with $\mathbb{E}[Y] = 0$; I require $b + Y \in \operatorname{dom}(V^*)$ almost surely for all relevant b.[2]

3 DARA preservation

The Arrow–Pratt measure of absolute risk aversion is $A_f(x) = {-f''(x)}/{f'(x)}$. A function f with $f' > 0$ and $f'' < 0$ exhibits *decreasing absolute risk aversion* (DARA) if A_f is strictly decreasing, which holds if and only if

$$f'''(x)\, f'(x) > f''(x)^2 \tag{2}$$

for all x in the domain.

[2]When V^* is CRRA this is a restriction on negative lifetime resources.

Theorem 1. *Let V^* be DARA. Then v defined by (1) is DARA under both multiplicative and additive risk. Strict DARA of v holds whenever V^* is not CARA. When V^* is CARA, v remains CARA under additive risk but becomes strictly DARA under multiplicative risk with non-degenerate Z.*

Proof. The proof chains two inequalities: a pointwise bound from DARA of V^* and a Cauchy–Schwarz inequality on expectations.

Additive case. With $h = b + Y,\ v^{(k)}(b) = \mathbb{E}[V^{*(k)}(b + Y)]$ for $k = 1, 2, 3$. DARA of v requires

$$\mathbb{E}[V^{*\prime\prime\prime}(b+Y)]\,\mathbb{E}[V^{*\prime}(b+Y)] \ \geq\ \big(\mathbb{E}[V^{*\prime\prime}(b+Y)]\big)^2. \tag{3}$$

DARA of V^* gives $V^{*\prime\prime}(x)^2 \leq V^{*\prime\prime\prime}(x)\,V^{*\prime}(x)$ pointwise. Since $V^{*\prime\prime} < 0$, this yields $-V^{*\prime\prime}(x) \leq \sqrt{V^{*\prime\prime\prime}(x)\,V^{*\prime}(x)}$. Taking expectations and applying Cauchy–Schwarz to $\sqrt{V^{*\prime\prime\prime}(b+Y)}$ and $\sqrt{V^{*\prime}(b+Y)}$:

$$\big(\mathbb{E}[-V^{*\prime\prime}]\big)^2 \ \leq\ \Big(\mathbb{E}\Big[\sqrt{V^{*\prime\prime\prime}\,V^{*\prime}}\Big]\Big)^2 \ \leq\ \mathbb{E}[V^{*\prime\prime\prime}]\,\mathbb{E}[V^{*\prime}],$$

which is (3).

Multiplicative case. With $h = bZ,\ v^{(k)}(b) = \mathbb{E}[Z^k V^{*(k)}(bZ)]$. From pointwise DARA, $Z^2(-V^{*\prime\prime}(bZ)) \leq Z^2\sqrt{V^{*\prime\prime\prime}(bZ)\,V^{*\prime}(bZ)}$. Cauchy–Schwarz applied to $Z^{3/2}\sqrt{V^{*\prime\prime\prime}(bZ)}$ and $Z^{1/2}\sqrt{V^{*\prime}(bZ)}$ gives

$$\big(\mathbb{E}[Z^2(-V^{*\prime\prime})]\big)^2 \ \leq\ \mathbb{E}[Z^3 V^{*\prime\prime\prime}]\,\mathbb{E}[Z\,V^{*\prime}].$$

Strictness. Overall strict inequality requires at least one of the two steps to be strict. The pointwise step is strict whenever V^* is not CARA, which suffices for strict DARA of v under either risk structure regardless of the shock distribution. When V^* is CARA, the pointwise step holds with equality. Under additive risk, the Cauchy–Schwarz step also holds with equality (since $V^{*\prime\prime\prime}/V^{*\prime}$ is then constant), so v is CARA. Under multiplicative risk with non-degenerate Z, the Cauchy–Schwarz proportionality condition fails, giving strict inequality and hence strict DARA of v. □

The result imposes no parametric restriction on V^*; the iso-elastic assumption in Abel and Warshawsky (1988) is not needed for DARA inheritance. The economic intuition is that $v'(b)$ is an average of marginal utilities across unresolved descendant states. Under DARA, marginal utility is a convex function whose curvature decreases with resources. Averaging across states preserves this structure because the Cauchy–Schwarz inequality disciplines how the dispersion of marginal utilities interacts with their level—the same mechanism that underlies the preservation of log-concavity under convolution.

4 Precautionary bequests

Having established that risk-aversion properties carry over from V^* to v, I now ask how increased descendant risk shifts the marginal value of bequests. A mean-preserving spread in the descendant's shock raises $v'(b)$ under standard conditions.[3]

Proposition 1. *Under additive risk, if $V^{*'''} > 0$ (prudence; Kimball, 1990), a mean-preserving spread in Y raises $v'(b) = \mathbb{E}[V^{*\prime}(b+Y)]$.*

Proof. $V^{*\prime}$ is convex under prudence. Apply Jensen's inequality. □

Proposition 2. *Under multiplicative risk, if $\phi(z) = zV^{*\prime}(bz)$ is convex in z—equivalently, if relative prudence $-xV^{*'''}(x)/V^{*''}(x) \geq 2$—then a mean-preserving spread in Z raises $v'(b) = \mathbb{E}[ZV^{*\prime}(bZ)]$.*

Proof. $\phi''(z) = b[2V^{*''}(bz) + bzV^{*'''}(bz)] \geq 0$ under the stated condition. □

For CRRA utility with coefficient γ, relative prudence is $\gamma + 1$, so Proposition 2 applies when $\gamma \geq 1$.

Proposition 1 provides the theoretical counterpart of the empirical finding in Boar (2021) that parental consumption responds to children's permanent income uncertainty. The result requires only prudence of the continuation value, a condition widely maintained in calibrated models.

5 Multiplicative risk: additional structure

Under multiplicative risk, the marginal bequest motive admits the decomposition

$$v'(b) = V^{*\prime}(b)\, \mathbb{E}\left[\frac{Z\, V^{*\prime}(bZ)}{V^{*\prime}(b)}\right], \tag{4}$$

separating a certainty-equivalent term from a risk adjustment (see Abel and Warshawsky, 1988, for a related HARA characterisation). When V^* is CRRA, as in many workhorse models, this imposes additional restrictions on the joy-of-giving bequest motive.

Proposition 3. *Under multiplicative risk with $V^*(x) = x^{1-\gamma}/(1-\gamma)$, the reduced form is $v(b) = \mathbb{E}[Z^{1-\gamma}] \cdot b^{1-\gamma}/(1-\gamma)$: CRRA with the same γ and a risk adjustment independent of b.*

Proof. $V^*(bZ) = Z^{1-\gamma}\, b^{1-\gamma}/(1-\gamma)$. Take expectations. □

The preservation of functional form relies on the homogeneity of pure CRRA utility; it does not extend to shifted power functions $V^*(x) = (x+\kappa)^{1-\gamma}/(1-\gamma)$ with $\kappa > 0$, where the interaction of κ with multiplicative scaling breaks the factorisation. Under shifted CRRA with $\kappa > 0$, the reduced form $v(b) = \mathbb{E}[(bZ+\kappa)^{1-\gamma}]/(1-\gamma)$ remains DARA by Theorem 1 but is no longer a member of the HARA class with the same parameters.

[3] A random variable $\tilde{Y}$ is a *mean-preserving spread* of Y if $\tilde{Y}$ has the same mean as Y and dominates Y in convex order, i.e., $\mathbb{E}[g(\tilde{Y})] \geq \mathbb{E}[g(Y)]$ for every convex g.

6 Additive risk: DARA without further structure

Under additive income risk, DARA is preserved and precautionary motives arise, but the additional structures that multiplicative risk delivers are absent.

Importantly, the functional form of descendant utility is not preserved. Under CRRA, $v(b) = \mathbb{E}[(b+Y)^{1-\gamma}]/(1-\gamma)$, which differs from $b^{1-\gamma}/(1-\gamma)$ whenever Y is non-degenerate. There is no factorisation of $v'(b) = \mathbb{E}[V^{*\prime}(b+Y)]$ that separates the bequest level from the distribution of Y: the shock enters inside $V^{*\prime}$, not through scaling. The ratio $v'(b)/V^{*\prime}(b)$ depends on b and converges to one as the bequest grows large relative to the support of Y.

This is the empirically prevalent case. Additive income risk is the specification in the Bewley–Huggett–Aiyagari tradition, in the dynastic precautionary savings framework of Boar (2021), and in most quantitative life-cycle models with bequest motives. In all such settings, assuming DARA for the reduced-form bequest motive is justified by Theorem 1. Assuming a specific functional form—CRRA, shifted CRRA, or otherwise—is a separate modeling choice that the underlying altruism does not discipline.

What does this mean in practice? The true reduced form under additive risk, $v(b) = \mathbb{E}[(b+Y)^{1-\gamma}]/(1-\gamma)$, has curvature that varies with b: absolute risk aversion is high at small bequests, where the additive shock is large relative to resources, and declines toward the CRRA benchmark as bequests grow. This pattern is naturally captured by the Stone–Geary or shifted-CRRA specifications commonly adopted in applied work—for instance, the $(b+\kappa)^{1-\gamma}/(1-\gamma)$ form in De Nardi (2004)—which generate declining curvature through a shift parameter κ. The connection is intuitive: κ plays a role analogous to the dispersion of Y, absorbing the effect of descendant risk into a single reduced-form parameter. The approximation is reasonable but necessarily incomplete, since the dependence of curvature on b in $\mathbb{E}[(b+Y)^{1-\gamma}]$ is shaped by the full distribution of Y, including its higher moments, in a way that a single parameter cannot fully represent. This offers a useful lens on cross-study variation in calibrated values of κ: such differences may reflect not only differences in the strength of the bequest motive but also differences in the descendant risk environment being approximated.

7 Concluding remarks

The practice of assuming DARA warm-glow bequests is consistent with the underlying altruism whenever the descendant's continuation value is itself smooth and DARA—conditions widely maintained in calibrated models. What the altruism does not supply is the functional form. Under multiplicative descendant risk, functional form is pinned down: pure CRRA continuation utility implies CRRA bequests. Under additive income risk—the dominant specification in quantitative work—it is not. The functional-form assumption carries independent empirical content and should be assessed against data on bequests and wealth, not inherited from the primitive by appeal to the altruistic motive.

In thes sense, reduced-form bequest preferences are ultimately estimated terminal value functions (e.g., Keane and Wolpin, 1994, 1997). Consequently, differences in calibrated bequest-motive parameters across studies may reflect not only differences in the strength of altruism but also differences in the descendant risk environment—or simply in the approximation error from imposing a particular functional form. The results here suggest that care is needed when mapping estimated or calibrated parameters to an underlying altruistic structure.

A Regularity

Differentiation under the expectation requires, for multiplicative risk, $\mathbb{E}[Z^k\,|V^{*(k)}(bZ)|] < \infty$ for $k \leq 3$; for additive risk, $\mathbb{E}[|V^{*(k)}(b+Y)|] < \infty$ for $k \leq 3$. The character of these conditions depends on the preference class. For CRRA under multiplicative risk, the binding requirement is typically $\mathbb{E}[Z^{1-\gamma}] < \infty$, a negative-moment condition when $\gamma > 1$ that restricts the left tail of Z. For CRRA under additive risk, the key requirement is that $b + Y$ is bounded away from zero almost surely. For CARA, exponential-moment conditions are needed. In all cases, the support of the shock distribution must be compatible with the domain of V^*.

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