Adverse Selection Among Early Adopters and Unraveling Innovation

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Abstract

I analyse "selection markets", where consumers differ in their willingness-to-pay and the costs they impose on sellers. Increased adverse selection among early adopters generates a dynamic free-rider problem—causing markets to unravel prematurely. With new products (e.g., reverse mortgages, insurance design) market power can mitigate inefficiencies stemming from adverse selection. Empirical evidence leverages differences in private information to show that early adopters in the market for deferred income annuities are more adversely selected. I then derive comparative statics ordering markets by their adverse selection, equilibrium prices, quantities, and welfare. These results explain three empirical phenomena: low uptake of existing products, slow demand for new products, and market inactivity despite unmet demand.

Keywords: Adverse selection, Insurance, Annuities; JEL Codes: D82, G22, D14

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Selection markets are ubiquitous. These are markets where consumers differ in the costs they impose on a firm when they purchase a good, including annuities, insurance, credit markets, financial securities, used goods, and even labor markets.[1]

Moreover, these markets are often characterized by low utilization, for example in annuities. Empirical work typically rationalizes this under-utilization through low consumer demand.[2] However, this explanation overlooks an essential puzzle: the supply side's failure to offer more attractive products despite unmet consumer demand. I show how adverse selection in immature markets exacerbates initial low demand and causes unraveling. Even if a market would be fully efficient if it could reach maturity, higher adverse selection among early adopters prevents it from maturing.

This paper's central conceptual point is to show that adverse selection in immature markets introduces a novel dynamic free-rider problem. When early adopters are more adversely selected, new markets can be unlikely to mature because this free-rider problem erodes any dynamic incentive to enter the market. Initial entrants bear the costs of adverse selection among early adopters, but subsequent entrants compete away long-run rents. This prevents any potential long-run rents, that could be used to cross-subsidize the initial losses, from realizing. Consequently, this initial adverse selection can result in permanent unraveling even when the mature market is efficient.

This rests on whether early adopters are more adversely selected than consumers in mature markets. Yet, the answer to this question is theoretically ambiguous. I formalize this argument with a mix of theory and evidence from the maturing market for deferred income annuities. First, I establish when adverse selection is unambiguously larger among early adopters in young selection markets. In immature markets, potential consumers have less confidence in the product and benefit less from taking up new

[1] Einav et al. (2021) provide an overview.

[2] See Lockwood (2012) and Pashchenko (2013) for annuities, Ameriks et al. (2016) and Lockwood (2018) in long-term care insurance, and both Nakajima and Telyukova (2017) and Cocco and Lopes (2020) in reverse mortgage markets. Alternatively see Einav et al. (2010) for results emphasizing adverse selection.

products. Consequently, demand in immature markets is lower than in mature markets. Crucially, demand falls, but selection patterns also change. Early adopters, those willing to buy the product despite their lower initial private valuations, are also the consumers that impose the highest costs on firms, making adverse selection generically higher in immature markets without resorting.

To understand descriptively how large adverse selection is among early adopters, and the contribution to equilibrium pricing, I exploit differences in how much private information is available for annuitants of different ages. Using hand collected data on annuity prices, this allows me to separate the role of adverse selection in the maturing deferred income annuity market from other strategic and price relevant factors. I find an economic and statistically significant decrease in adverse selection as the market matures. Consistent with adverse selection, groups with larger private information experience larger reductions in equilibrium prices. Thus, adverse selection among early adopters helps explain a puzzling lack of innovation.

I thus provide a *joint* explanation for three important empirical regularities in selection markets: first, the low take up of existing products, second, sluggish demand when new products are introduced, and, third, market inactivity despite substantial unmet demand for new or modified alternatives. For example, Zinman (2014) highlights missing rungs in the consumer lending ladder, while Ameriks et al. (2016) documents unmet demand for a better formulation of long-term care insurance. Similarly, Cocco and Lopes (2015) establish demand for alternative reverse mortgage products, while Michaud and Amour (2023) document potential demand for risk management products that bundle existing insurance. Yet, Nakajima and Telyukova (2017) and Webb (2011) document very low take up rates for novel reverse mortgage and annuity products, respectively. Similarly, despite being sold to the public since 1954, the market for variable annuities (bundling tax-favorable saving, insurance, and indexed annuity options) remained small until the early 1990s (Brown and Poterba 2006).[3]

This paper's first contribution is to outline the implications of this novel free rider problem in a simple and intuitive framework. A key insight delivered by this approach is that even would-be efficient markets can unravel during their immaturity and fail to mature. This is a new, dynamic source of market failure building on the classic insight of Akerlof (1970). I show that unraveling in immature markets gives rise to a free-rider problem – leading to permanently dormant markets. Even though firms have dynamic incentives to enter profitable mature markets, firms may be unwilling to bear the cost of familiarizing consumers with new products in adversely selected markets because other firms compete away profits once it reaches maturity. This happens even though firms know lower willingness-to-pay and increased adverse selection in immature markets is transitory. These results explain the empirical phenomena of missing selection markets even when evidence suggests adverse selection in maturity may be limited.

I then show that the existence of the free rider problem challenges the common understanding of the interplay between market power and adverse selection, a relationship typically analyzed within static frameworks. Traditional static analyses, for example Starc (2014) and Mahoney and Weyl (2017), highlight that adverse selection can mitigate the negative impacts of market power by constraining firms' ability to extract rents. Nevertheless market power exacerbates under-supply. The simple dynamic analysis, however, reveals a mechanism that works in the opposite direction: market power can, in fact, alleviate dynamic inefficiencies induced by adverse selection. It allows a monopolist to cross-subsidize early losses from adverse selection with longer-run gains. Thus, in this dynamic setting market power becomes a tool to constrain the social cost of adverse selection – reversing implications for policy design.

[3] More generally, concern about missing innovation in insurance markets dates back over half a century: Rudelius and Wood (1970) identify heterogeneous adoption of life insurance innovations, Dorfman (1971) provides evidence of research activity and barriers to innovation, and Murray (1976) surveys insurance companies to identify innovations.

Having identified this novel source of market failure, I also consider the role of government intervention in promoting entry and innovation within missing selection markets. I discuss how increasing patent lengths or burdensome regulation can serve as an effective, time-varying subsidy with minimal informational requirements. This solution addresses two fundamental challenges: the inherent absence of data in missing markets and the potential distortions from intervention in mature markets.

Ultimately, however, increased consumer sorting is an empirical question. I next turn to an empirical analysis of early adopters which establishes increased adverse selection among early adopters: an important empirical contribution. This allows me to test the implications of the stylized model as well as to learn descriptively about the impact of market maturity on adverse selection. Using hand collected panel data on life-insurer pricing in the market for deferred income annuities, I identify selection separately from changes in other price relevant factors or strategic behaviour by exploiting differences in private information across age groups. This determines the propensity for adverse selection. As cumulative past sales increase in this immature market, I find that pricing for groups with more private information falls by more than groups with less private information. This approach identifies the selection channel separately from other pricing dynamics and establishes that adverse selection is larger among early adopters. Thus, in equilibrium adverse selection falls in a growing, or maturing, market. This is consistent with Warshawsky (1988) who documents declining adverse selection costs for annuities sold to women over the 20th century. Although it is not possible to test hypothetical dynamics in markets that do not exist, I show that the model predictions are empirically relevant.

The paper's fourth contribution is technical and provides the building blocks to understand the important empirical phenomena outlined above. In doing so, I establish a set of general comparative statics results for selection markets with the same underlying distribution of cost-relevant types under an assumption of monotonicity. This allows me to microfound the payoffs that generate the new dynamic free-rider problem. However, monotonicity is not guaranteed in problems with multidimensional heterogeneity (Fang and Wu 2018). Thus I characterize comparative statics using the joint distributions of model primitives. This allows me to develop an ordering over adversely selected markets in terms of their equilibrium quantity, prices, selection, and welfare. Moreover, I show this in an environment that admits a graphical representation, providing sufficient conditions for a graphical analysis of the selection problem with multi-dimensional heterogeneity. This is a key contribution of the paper.

I provide explicit and economically interpretable sufficient conditions on joint distributions which characterize shifts in the demand and cost curves in selection markets. These conditions additionally provide sufficient conditions for an ordering over equilibrium adverse selection, demand and welfare between markets. The advantage of the non-parametric conditions is that it admits a variety of commonly used specifications of consumer payoffs. In adversely selected markets under-insurance rises and consumer surplus falls when sorting on cost increases or private valuations decrease. I apply these comparative static results to characterize differences between mature and immature markets, nevertheless the insights apply equally to markets indexed by space and time or for different products.

I establish these results in a standard model of insurance demand with multidimensional heterogeneity. While I focus on insurance markets to provide a concrete example, results throughout the paper apply to selection markets in general. In this framework, consumers are heterogeneous along multiple dimensions including costs and risk premia, which determine the surplus from acquiring insurance, as well as other heterogeneity which captures frictions or departures from rationality. I consider adverse selection settings in which consumers' willingness-to-pay for insurance is positively correlated with their costs to insurers.[4]

[4] Analogous results can be obtained for advantageous selection.

To study unraveling and entry in novel markets, I focus on mature and immature markets. Mature markets are those that have existed for a long time and are well understood by consumers, while immature markets are new and unfamiliar. The standard framework can be extended to capture the differences between mature and immature markets by allowing consumers' willingness-to-pay for the product to differ across market maturity.

These differences in willingness-to-pay capture objective differences, such as short-term counterparty risk (Briggs et al. 2023) or initial self-insurance patterns that consumers' cannot quickly adjust, which lower the value of purchasing insurance. However, they can also capture subjective differences including consumers' initial perceptions or distrust of new products, for example inertia (see Handel 2013, who studies health insurance) or learning dynamics (see Israel 2005, for the case of learning in autmobile insurance). These differences between mature and immature markets lower demand and increase consumer sorting based on costs. This increases adverse selection in immature markets and generates under-insurance in equilibrium with lower adoption and higher costs.

I frame these results in the context of market maturity and use them to compare mature and immature selection markets. Comparative statics reveal that, under reasonable assumptions on the primitives of immature markets, demand is lower and the cost to serving the market increases for insurers compared with mature markets. This rationalizes sluggish demand for new products while also predict the low utilization of existing products and the complete unraveling of these markets.

Related Literature

I show that a novel free-rider problem manifests in the dynamics of adversely selected markets. Importantly, this reverses standard intuition for the interaction of market power and adverse selection. At any point in time, adverse selection impacts firm's ability to exploit market power as discussed by Starc (2014) and Mahoney and Weyl (2017). Moreover, market power exacerbates the under-supply driven by adverse selection. Here, however, the presence of market power early in a product's life cycle can incentivize entry when would-be efficient markets fail to mature.

I examine the implications of changing adverse selection over time and study policies promoting initial market entry. These results emphasize that inter-temporal cross-subsidization can be optimal under missing markets. These policies are related to, but differ from, those designed to mitigate selection (Geruso and Layton 2017, review these policies in health insurance markets).

To understand how these dynamic incentives can be microfounded, I develop comparative static tools that provide non-parametric conditions for an ordering over partial unraveling in Akerlof markets and are used to consider the dynamic implications of firm entry decisions in novel selection markets. This non-parametric characterization allows the theoretical insights to map onto commonly used empirical specifications, for example the constant absolute risk aversion utility used by Cohen and Einav (2007). Akerlof (1970) studies the failure of price mechanisms and market unraveling in selection markets. An extended no-trade theorem for Rothschild and Stiglitz (1976) markets is provided by Hendren (2013), while Attar et al. (2021) provide necessary and sufficient conditions for no-trade theorems in adverse selection markets.[5] Kong et al. (2023) show how adverse selection contributes to non-entry among differentiated firms, even absent fixed costs. This paper extends non-entry results to cases where adverse selection declines over time.

I study changes in the distributions of choice frictions similarly to Handel et al. (2019). I provide comparative statics for non-local changes in both the distribution of choice

[5] A sizable literature studies endogenous contract terms under adverse selection (e.g. Rothschild and Stiglitz 1976; Handel et al. 2015; Veiga and Weyl 2016; Azevedo and Gottlieb 2017) or screening with both intensive and extensive margins of choice (Geruso et al. 2023). This is not the focus of this paper.

frictions and the insurance value of contracts across markets. I then use this approach to build a partial ordering of equilibrium prices, quantities and welfare in terms of model primitives. Handel et al., however, study local reforms (including information interventions) that alter consumer's choice frictions and provide sufficient statistics formulas for their welfare impacts. As in Handel et al. and Spinnewijn (2017) I show how changes to model primitives lead to the rotations and shifts to cost and demand that are studied in other contexts (e.g. Starc 2014; Mahoney and Weyl 2017).

1. Will Deep Pocketed Insurers Enter When Immature Markets Unravel?

I begin by studying how adverse selection among early adopters leads to would-be efficient markets failing to mature. To answer this, I illustrate the key economic forces determining insurer entry into new markets using a two firm, two period model. I show that adverse selection produces a novel form of dynamic free-rider problem which makes new markets difficult to grow.

When adverse selection is severe enough, even the promise of market power is not enough to entice insurers to enter. Despite the fact that mature markets may be efficient, *temporary and predictable* increases in adverse selection give rise to missing markets. Therefore, the existence of this free-rider problem explains the absence of better insurance products including redesigned long-term care insurance or reverse mortgage products.

These results reverse the standard intuition for the interaction between market power and adverse selection derived in static frameworks. In static analysis (e.g. Starc 2014 or Mahoney and Weyl 2017 when quantity is low), adverse selection ameliorates the problems of market power by limiting the extent to which firms can earn rents by abusing market power. Market power, however, also exacerbates under-supply. In contrast, this dynamic setting illustrates how market power can ameliorate dynamic inefficiencies stemming from adverse selection because monopolists can cross-subsidize early losses with longer-run gains. This is in contrast to the implications of a static analysis – highlighting the complexity of policy intervention in selection markets.

In each period, firms choose whether to offer an insurance product taking as given contract terms. They observe entry decisions and post prices, competing a la Betrand. Consequently, prices equal average costs if both firms enter (e.g., equation 6 below). If, however, only one firm enters, they are able to exploit their market power and equate marginal revenue with marginal cost, delivering the usual monopolist pricing rule.

To close the model, I assume the market begins in an immature state. This state of the world updates from immature to mature in the second period if and only if at least one firm enters in the first period and sells a positive quantity.[6] This tractable framework captures the interaction between market power, adverse selection, and pricing, as well as the investment required for a market to reach maturity.

The following proposition characterizes how market immaturity interacts with market forces and can endogenously give rise to market incompleteness in this setting. The proof is in Appendix A.

PROPOSITION 1. *If a monopolist cannot earn positive profits in immature markets, there exists a subgame perfect equilibrium where no firm enters. Moreover, this is the unique extensive-form trembling hand perfect equilibria.*

Importantly, the statement depends only on primitives of the immature market and, thus, holds even when the mature market is efficient. In other words, static adverse selection in new markets can lead to missing mature markets regardless of whether the mature market is efficient.

[6] Consumers learn about the generic "product" offered by firms instead of developing brand loyalty.

The intuition for the result is simple, but reveals a new free-riding problem preventing insurance markets from maturing. Consumers are adversely selected. When early adopters are more adversely selected, a firm entering into an immature market is effectively investing (by earning negative profits) in the market reaching maturity.[7] If they could guarantee monopolist profits in the mature market, it may be profitable to absorb the temporary hit to their bottom line. Herein lies the free-rider problem. The entrant pays a cost to reduce future adverse selection for all firms. For example, by familiarizing consumers with the new product. However, another firm always has an incentive to wait and enter the mature market once their competitor pays this cost. The original entrant pays the costs of the initial selection, but later entrants compete away any potential long-run rents that could be used to cross-subsidize the initial losses. Consequently, no firm is willing to enter the immature market.[8]

This logic rests on the possibility that no positive profits exist in the market's infancy. Even for a monopolist, there is no guarantee positive profits exist under adverse selection. To see this, consider the following standard expression for the insurer's profit margin (Starc 2014; Kong et al. 2023):

$$p - AC(p) = \frac{1}{\eta_{D,p}} - (AC(p) - MC(p)) \,. \tag{1}$$

The first term on the right hand side is the usual Lerner markup. The second term captures the influence of adverse selection on firm pricing through their desire to "cream skim" low cost consumers at the margin. This demonstrates how adverse selection can discipline market power (Mahoney and Weyl 2017, highlight that this effect is reversed at high quantities). However, firms may earn negative monopolist profits in the absence

[7] Indeed, at least one major Canadian auto-insurer offers individuals a subsidy (above rating incentives) to enrol in new contracts using telematic monitoring.

[8] This implicitly assumes that the aggregate state transitions only once an amount of contracts exceeds a positive threshold, thus firms cannot 'enter' by charging a price that guarantees zero-profits and no contracts sold.

of fixed costs. This happens when either the price elasticity of demand is large, so $1/\eta_{D,p} \approx 0$, or the downward pressure of adverse selection is severe enough.[9]

Implications For Market Intervention: Patents or Burdensome Regulation as Time Varying Subsidies. If there is surplus in mature markets, it may be socially optimal for a market to mature even if there is no-trade in its infancy. However, permanent policy solutions, like a subsidy, may be inefficient because the primitives evolve over time. For instance, in the entry and exit game above there is a welfare cost associated with missing mature markets. Yet, once the market reaches maturity the competitive allocation can be efficient.

In this case, phasing out subsidies over time when there is deadweight loss from government intervention in mature markets is optimal, but implementing an optimal time varying subsidy may be infeasible due to the information requirements for the social planner. Tautologically, there is no choice data to observe if a market is missing. Therefore, it is hard to conjecture how the planner would set a subsidy optimally.

A simple decentralized alternative policy is to remove barriers to patenting or to increase the length of patent protection in insurance markets. Intuitively, this raises the monopolist profit in immature markets by protecting firms against free-riders and encourages entry. It offers an effective time-varying subsidy. In principal, this simplifies the policy problem because it now depends only on patent length (as opposed to a time varying subsidy schedule). However, there still remains an information problem: the planner needs to have enough information to select an optimal patent length.

One advantage of patents over subsidies is that the filing of patents requires information disclosure on the part of firms. This, in principal, can be used to collect

[9] Kong et al., similarly discuss entry under imperfect competition and derive a threshold condition on the slope of the average cost curve to sustain a given market structure.

information which can be used in determining the optimal patent length.[10] In contrast, in the absence of the insurance market, a planner relying on subsidies has to will the insurance product into existence. This involves choosing from an abstract menu of hypothetical insurance ideas and hoping that it is technologically feasible. In contrast, the patenting planner is able to choose from a menu of observed innovations.

An alternative policy tool is to increase the regulatory burden on insurers (or comparable institutions in other selection markets) who offer new products. For example, regulatory agencies can require extensive disclosure requirements or mandate costly reporting of newly issued policies. This increases the cost of adopting existing technology and slows diffusion across firms (Benhabib et al. 2021).[11] This slowed diffusion lengthens the period of market power.

2. A Model of Competitive Equilibrium in the Insurance Market

The previous section establishes that adverse selection can give rise to a new form of free-rider problem. This dynamic inefficiency illustrates how market power can improve market outcomes by allowing the monopolist to cross-subsidize early losses with longer-run gains. I now show how the evolution of willingness-to-pay and consumer sorting generate the payoffs at each phase of maturity that give rise to dynamic free-riding. To do so, I first present a canonical model of insurance contracts. Subsequently, I discuss how differences between mature and immature markets lower demand and increase consumer sorting based on costs. I show this increases adverse selection in immature markets and generates under-insurance in equilibrium with lower adoption and higher costs. Thus, the differences between mature and immature markets can lead to the unraveling of competitive equilibria when market immaturity exacerbates

[10] See Williams (2017) for a recent survey of other relevant considerations in the optimal design of patent length and scope.

[11] Empirical evidence highlights the role of labor regulation in suppressing innovation. For example, Griffith and Macartney (2014) and Aghion et al. (2023).

adverse selection.

2.1. Demand, Supply, and Adverse Selection

I consider an Akerlof (1970) model of insurance in the style of Einav et al. (2010) and Einav and Finkelstein (2011). A single perfectly competitive and risk-neutral insurer sells insurance to a unit continuum of heterogeneous individuals at a common price p. The terms of the insurance contract are exogenous and common across all individuals.

Demand. Individuals are heterogeneous along multiple dimensions. Namely, their risk type π_i which determines the expected cost to the insurer, their risk premium r_i, and potential demand frictions ε_i. These are private information for individual i.[12] An individual buys insurance if and only if their *subjective* willingness-to-pay, defined as

$$\omega_i \equiv u(\pi_i, r_i, \varepsilon_i) = u(\underbrace{\pi_i + r_i}_{\textit{insurance value}} + \varepsilon_i), \tag{2}$$

for any strictly increasing utility function $u(\cdot)$, exceeds the price of the insurance. In addition to their risk type, individual i's risk premium, r_i, determines the surplus in the insurance market.

Following Spinnewijn (2017), individual choices depend on additively separable demand frictions, ε_i, which can distort insurance demand. This approach is agnostic about the nature of demand frictions, nesting a number of important model features including borrowing constraints, but also inertia, subjective beliefs, and bounded rationality. These frictions are heterogeneous and can lead individuals to overvalue ($\varepsilon > 0$) or undervalue ($\varepsilon < 0$) the insurance contract. As risk premia and frictions have identical effects on consumer choices I focus on consumers' perceived risk premia, net

[12] Conditioning on observable characteristics correlated with π_i may be illegal. Nevertheless, this can be interpreted as a model of insurance contracts in each partition of the observable characteristic space.

of any behavioral influences, given by $\tilde{r}_i \equiv r_i + \varepsilon_i$, for the majority of the analysis.

Thus, demand is given by

$$D(p) = 1 - F_\omega(p) = \mathbf{1}[\omega > p], \tag{3}$$

where F_x denotes the CDF of variable x.[13]

Supply. The cost of providing insurance at price p is determined by the risk types of individuals purchasing insurance at that price.[14] Consequently, *average* and *marginal* costs are given by:

$$AC(p) = E(\pi|\omega \geq p) \tag{4}$$

$$MC(p) = E(\pi|\omega = p). \tag{5}$$

When willingness-to-pay, ω, is increasing in the cost to the insurer, π, there is adverse selection; marginal costs increase in price and the MC curve is downward sloping.

Equilibrium. To simplify notation, I assume demand and cost curves are continuous and monotonic which implies the equilibrium is unique if it exists. The equilibrium price is given by the break even price that pools expected costs

$$p^\star = p : D(p) = AC(p). \tag{6}$$

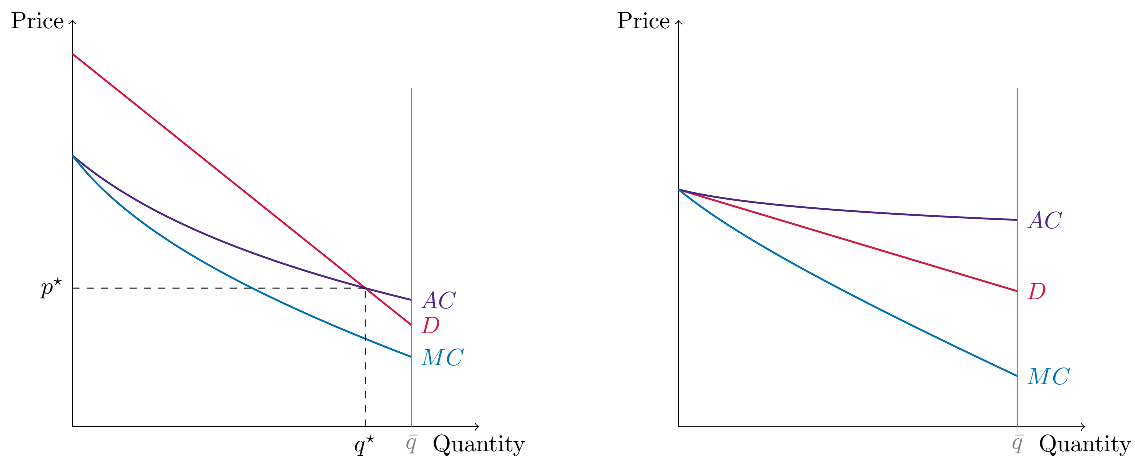
Consequently, the equilibrium allocation may differ from an efficient allocation. An efficient allocation insures everyone whose willingness-to-pay exceeds their expected cost ($\omega_i > \pi_i$), those for whom the demand curve lies above the marginal cost curve as

[13]Similarly, μ_x and σ_x^2 will denote the mean and variance of x's distribution.

[14]For simplicity, I abstract from administrative loadings, or fixed costs which don't affect the main result.

shown in Panel (A) of Figure 1.

Furthermore, a positive equilibrium price does not necessarily exist. The insurance market unravels (Akerlof 1970) when individuals are not willing to purchase insurance at the pooled cost of those with higher demand. This occurs when the demand curve lies entirely below the average cost curve (Hendren 2013) as shown in Panel (B) of Figure 1.



(A) Textbook Adverse Selection

(B) Textbook Unraveling

FIGURE 1. Equilibrium in the Insurance Market Without Demand Frictions

2.2. Comparative Statics Between Markets and Unraveling in Immature Markets

I now turn to understanding the implications of market maturity for adverse selection and market unraveling. To do so graphically, I focus on cases where both demand and average and marginal cost curves are monotone. This reasonable restriction is common in a large body of empirical work following Einav et al. (2010) and Einav and Finkelstein (2011) (Einav and Finkelstein 2023 provide a recent review of the literature adopting this framework). I provide conditions on the primitives of the model which give rise to monotonicity in Section 4 and formally state the assumptions on mature and immature markets in the context of the general framework. Note that monotonicity does not require that choice frictions are independent of other characteristics (Solomon (2023) finds evidence in favour of dependence). Consequently, choice frictions are a reduced form representation of a richer economic model compatible with a range of behavioral microfoundations. This includes rational inattention models of endogenous information acquisition and settings in which agents make correlated mistakes affecting both risk premia and insurance decisions (Leive et al. 2022).

To make the markets comparable, I hold the distribution of potential and realized risks constant across mature and immature markets. The key idea is that these are mature and immature markets for an identical insurance product with identical potential and realized risk pools.[15] The second assumption I make is that willingness-to-pay is depressed in immature markets relative to mature markets – lowering demand. Lower demand can stem from either differences in the objective risk premia, r, or demand frictions, ε, and I begin by motivating differences in the first component of net premia, $\tilde{r}$.

Three channels generate objective differences between the risk premia for immature and mature markets. First, individual portfolios may not be optimized to take advantage of an insurance product when it is first introduced. For example, an individual may accumulate savings to self-insure in the absence of formal insurance. Thus, the benefits to insurance purchase are larger when other choices can adjust to fully capitalize on the product. Second, there may be objective uncertainty about the probability that all claims will be covered, the hassle costs associated with claiming, or the likelihood of reclassification risk. For example, in health insurance markets, there is uncertainty over, and heterogeneity in, the coverage of new drugs or procedures. This uncertainty is likely larger for new types of insurance and lowers the risk premia. Third, objective counterparty risk may be larger relative to mature markets where insurers typically trade with repeated cohorts of potential insurees. In an immature market insurers

[15] Thus, neither firms nor individuals can alter the distribution of risks in mature markets.

receive liquid assets now (in the form of premia), but may not make payouts until far in the future (for example, life insurance, deferred annuities, or long term care insurance). This difference in the timing of liquidity and liabilities may distort the insurers portfolio choice (Thompson 2010, models endogenous counterparty risk) and raise the possibility of default or non-payment in immature markets – leading to lower risk premia.

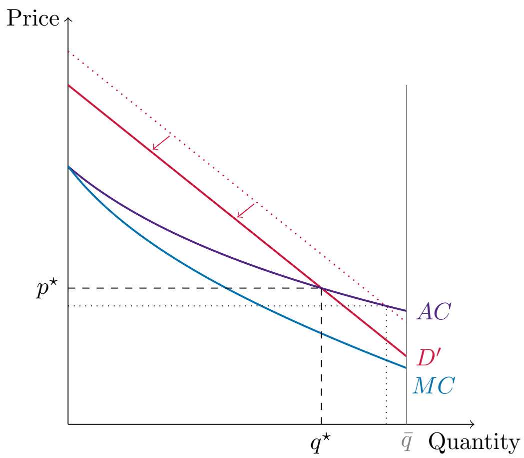
Instead, lower demand may be the result of demand frictions that are systematically more negative in immature markets. Status quo biases, distrust of new products, or inertia costs all imply smaller biases against products in mature markets compared with immature markets. As does lack of awareness (Boyer et al. 2020) or when new products are less likely to enter consideration sets. Similarly, negative demand frictions that disappear as consumers learn or experience the product[16] can be represented in the same way. In this setting, immature markets have *more negative* demand frictions in contrast to mature markets where a larger fraction of consumers have been exposed to a product in the past through peers, advertising, or past purchase.

This has an important implication for demand: at any given price, the average individual has a larger willingness-to-pay in mature markets and consequently more individuals choose to purchase insurance. Panel A of Figure 2 displays the implications of this *level effect* on demand.

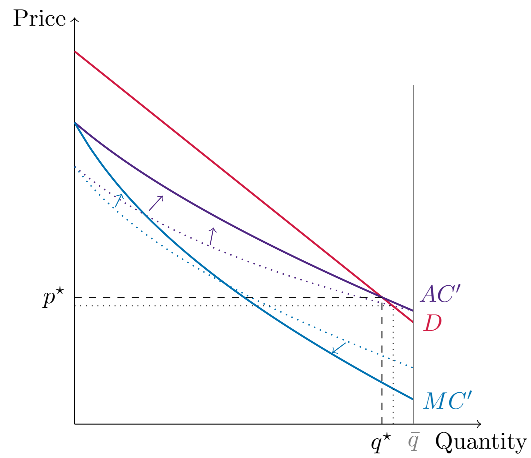
This level effect on demand holds average and marginal costs constant for any given quantity. Although households need lower prices to induce purchase, the ordering of consumers willingness-to-pay is unchanged. Therefore the identity of marginal (and inframarginal) consumers at any quantity demanded is the same and, as Panel A shows the equilibrium price increases and demand falls (See Proposition 2).

Early adopters can differ from the population of consumers who purchase in mature markets in other important ways and, in addition to having lower demand, they may

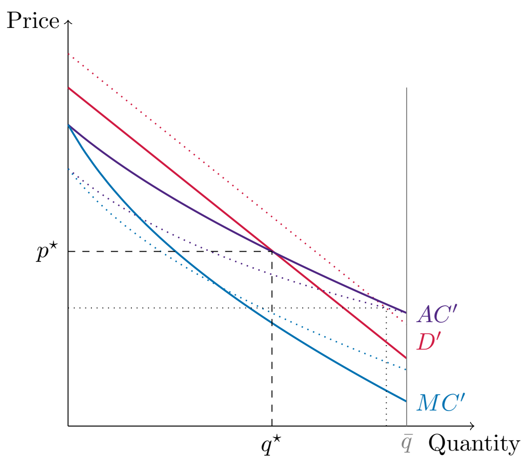
[16]Evidence on tax minimization (Chetty et al. 2013) and retirement plan participation (Duflo and Saez 2003) suggests peer experience matters.



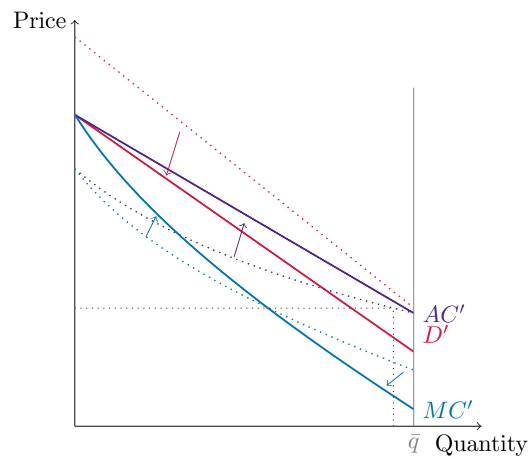
(A) Demand Contraction (Level Effect)



(B) Increased Adverse Selection (Sorting Effect)



(C) Under-insurance in the Short Run



(D) Unraveling in the Short Run

FIGURE 2. The Long and Short Run of Insurance Markets

sort more strongly on price.[17] Then, the slope of the marginal cost curve is steeper and they are more adversely selected as captured by the marginal cost of insurance.

Intuitively, in this case, those who have the highest willingness-to-pay for insurance impose the highest costs on insurers for their coverage – making early adopters uniformly more costly on average. Crucially, although average costs increase at any

[17]This is ultimately an empirical question. Section 3 provides an example where market equilibria supports increased sorting on price and Section 4 discusses this further in the context of formalizing the assumption.

price when there is a level effect, a change in the sorting patterns of individuals is necessary to induce a change in the cost curves insurers face conditional on quantity. When consumers are reordered and selection on cost increases, insurers face steeper marginal cost curves and also face higher *average* costs.

Suppose individuals were ordered by their willingness-to-pay. Without re-sorting, this ordering is identical in mature and immature markets. Thus when q individuals purchase insurance, the marginal consumer is identical (q along this order) and imposes identical expected costs on the insurer. If, instead, there is re-sorting, individuals are reshuffled and the identity of the marginal consumer changes. Increased sorting on cost in immature markets, relative to mature markets, results in the purchase of insurance signaling higher average costs in the immature market.

Panel B of Figure 2 shows this outward expansion of the average cost curve in immature markets.[18] Even holding the level of demand constant, the increase in adverse selection among the inframarginal consumers of the insurance product drives up expected costs for the insurer. Consequently, insurers set higher prices to cover the cost of providing insurance, even though some lower cost individuals select out of the pool. Thus, the equilibrium price increases and quantity falls. There is (additional) partial unraveling in the market.[19]

The following results formalize the implications of market maturity on the market equilibrium. Importantly, they rely only on the assumption of monotonicity and the role of demand contractions and increased sorting.

PROPOSITION 2 (Partial Unraveling). *If an equilibrium exists, the equilibrium price is higher in immature markets and fewer consumers are insured,* $q^{MM} \geq q^{IM}$ *and* $p^{MM} \leq p^{IM}$.

[18] This is a rotation of the average cost curve around full insurance as in Mahoney and Weyl (2017). The difference in average costs in (36) is decreasing in quantity demanded, reaching zero at full coverage. Intuitively, at full coverage the entire population is insured and $AC^{IM}(1) = AC^{MM}(1) = \mu_\pi$. In contrast, Starc (2014) considers rotation around equilibrium demand.

[19] This can occur even if marginal costs in immature markets fall below demand which, absent choice frictions, implies full coverage is efficient.

PROOF. Let p^{MM} be the equilibrium price in the mature market,

$$D^{MM}(p^{MM}) = AC^{MM}(p^{MM}) \ . \tag{7}$$

If it is also an equilibrium price in the immature market, then

$$D^{IM}(p^{MM}) = AC^{IM}(p^{MM}) \tag{8}$$

also holds. But the demand contraction and increased sorting[20] imply the following inequality must also hold

$$D^{IM}(p^{MM}) \leq D^{MM}(p^{MM}) = AC^{MM}(p^{MM}) \leq AC^{IM}(p^{MM}) \ , \tag{9}$$

which is a contradiction unless the immature and mature markets are identical.

Furthermore, if there exists a price $p^{IM} \neq p^{MM}$ that clears the immature market then differences across markets satisfy *strict* dominance. Therefore

$$D^{IM}(p^{IM}) = AC^{IM}(p^{IM}) \longleftrightarrow p^{IM} > p^{MM} \ , \tag{10}$$

and

$$q^{IM} = D^{IM}(p^{IM}) < D^{MM}(p^{IM}) < D^{MM}(p^{MM}) = q^{MM} \ . \tag{11}$$

□

Panel C of Figure 2 illustrates the new equilibrium graphically. Marginal costs are below demand in both mature and immature markets, thus full insurance is the efficient outcome in both regimes. Despite this, there is additional under-insurance relative to

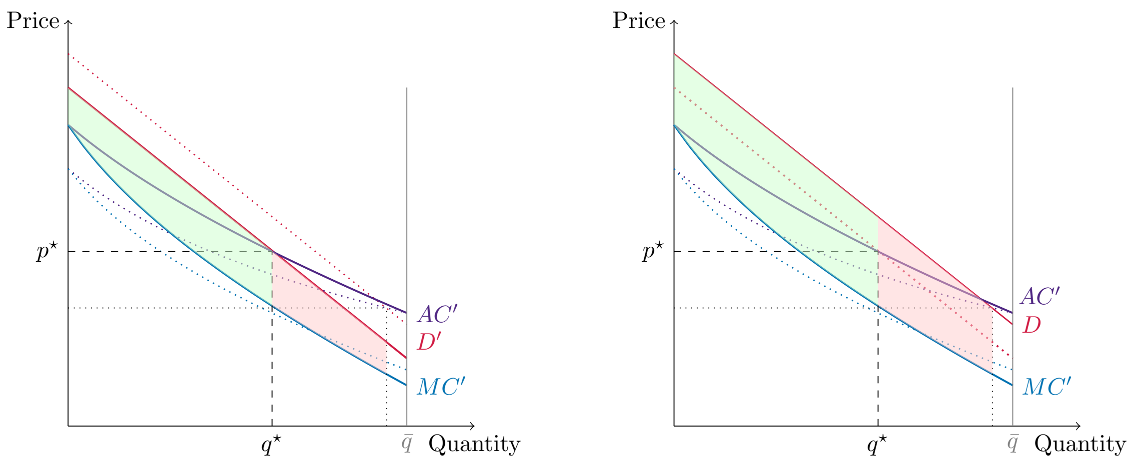
[20]This will be formalised in the general case as Propositions 5 and 7.

the mature market due to both the level effect of demand contraction and the increased adverse selection due to additional sorting. The increase in equilibrium price and reduction in quantity demanded is larger than either effect in isolation (Panels A and B).

An alternative consequence of immaturity is that markets may unravel.

COROLLARY 1 (Complete Unraveling). *Would-be efficient mature markets can unravel before they reach maturity.*

This happens when the average cost curve lies above demand for all prices (Panel D of Figure 2). Thus, even a market that would be efficient if it could mature can unravel in its early stages. As Section 1 shows, this can lead to permanently missing markets.



(A) Revealed Preference Benchmark

(B) Paternalistic Benchmark

FIGURE 3. Welfare Losses in Immature Insurance Markets

The presence of under-insurance ($q^{IM} < q^{MM}$) does not depend on whether changes in willingness-to-pay are driven by shifting risk premia or choice frictions. Calculating welfare costs, however, require a stance on their relative importance and the source of the change. The following proposition evaluates welfare under two policy relevant benchmarks, which are illustrated in Figure 3.

Proposition 3 (Efficiency Costs). *Consumer surplus in immature markets is lower at the immature market equilibrium than the mature market equilibrium ($CS^{IM}(q^{IM}) \leq CS^{IM}(q^{MM})$). This is true irrespective of whether demand for insurance is evaluated at:*

i. Willingness-to-pay in immature markets (revealed preference benchmark)

ii. Willingness-to-pay in mature markets (paternalistic benchmark)

Thus, the equilibrium in immature markets is pareto dominated by the mature market.

Proof. First, note that firms always earn zero profits in equilibrium. Consequently, consumer surplus is sufficient to pareto order equilibria.For a demand level $q^{\star}$, consumer surplus in immature markets is defined as

$$CS^{IM}(q^{\star}) = \int_0^{q^{\star}} \tilde{D}^{IM}(q) - MC^{IM}(q)\, dq \qquad \text{where} \qquad \tilde{D}(q) = D^{-1}(q), \tag{12}$$

under the revealed preference benchmark. It follows that the difference in consumer surplus due to increased adverse selection is

$$CS^{IM}(q^{MM}) - CS^{IM}(q^{IM}) = \int_0^{q^{MM}} \tilde{D}^{IM}(q) - MC^{IM}(q)\, dq - \int_0^{q^{IM}} \tilde{D}^{IM}(q) - MC^{IM}(q)\, dq, \tag{13}$$

and given $q^{MM} \geq q^{IM}$ (Proposition 2) this can be expressed as

$$CS^{IM}(q^{MM}) - CS^{IM}(q^{IM}) = \int_{q^{IM}}^{q^{MM}} \tilde{D}^{IM}(q) - MC^{IM}(q) \geq 0, \tag{14}$$

which captures the expanded coverage of insurance in mature markets. Figure 3 Panel A illustrates this graphically where the red shaded area to the right of $q^{\star}$ denotes the foregone surplus due to the additional under-insurance in the short run – the area between the marginal cost curve and the immature market demand curve.

For the paternalistic benchmark which uses the mature market inverse demand

$$CS^{IM}(q^{MM}) - CS^{IM}(q^{IM}) = \int_{q^{IM}}^{q^{MM}} \tilde{D}^{MM}(q) - MC^{IM}(q) = \tag{15}$$

$$\underbrace{\int_{q^{IM}}^{q^{MM}} \tilde{D}^{MM}(q) - \tilde{D}^{IM}(q)}_{\text{Paternalism } (\geq 0)} + \underbrace{\int_{q^{IM}}^{q^{MM}} \tilde{D}^{IM}(q) - MC^{IM}(q)}_{\text{Case (i)}} \geq 0, \tag{16}$$

where the positive paternalistic contribution follows from the contraction of demand (Proposition 5 in the general analysis below). Figure 3 Panel B shows this graphically. □

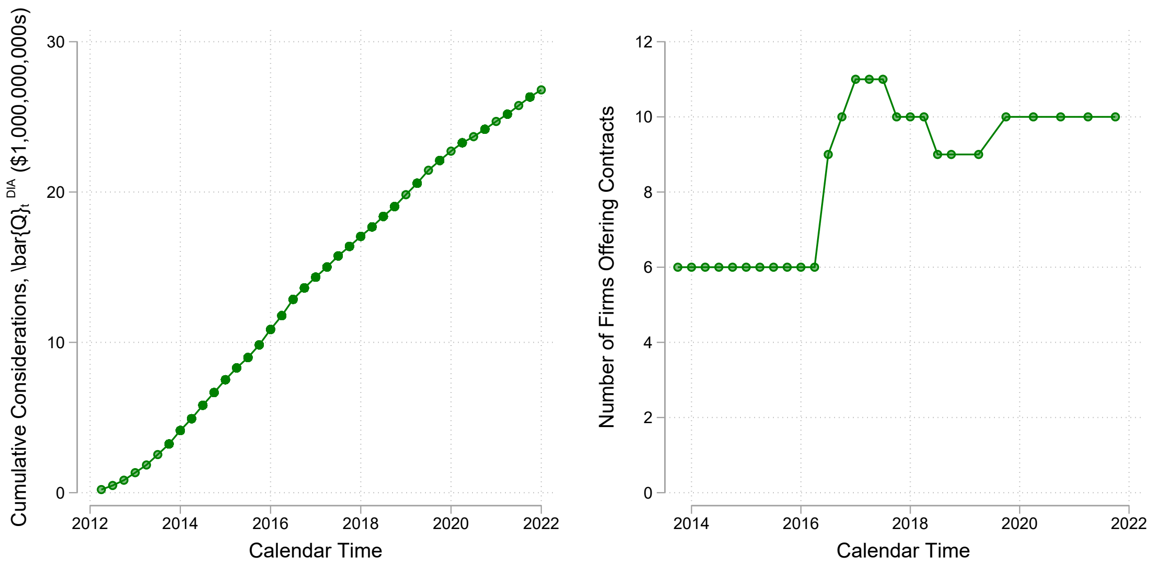
3. Identifying Adverse Selection Among Early Adopters: Evidence from the Market for Deferred Income Annuities

I now provide evidence that early adopters are more adversely selected. As there is no data on missing markets, I instead study the comparison between different stages of market maturity and show how they align with testable implications outlined above. Specifically, I focus on deferred income annuities – a product similar to a traditional annuity, but where payouts do not begin until a date (or age) agreed in the future. Hence, they provide deferred income.

Data on the aggregate size of the market, as measured by the dollar value of new annuity considerations, is collected from the quarterly reports of the Life Insurance Marketing and Research Association.[21] Figure 4A shows how cumulative sales evolved over time. At the start of this period the market for deferred income annuities was small, totaling around $200 million and accounting for less than 1% of the overall fixed income

[21] Specifically, the U.S. Individual Annuity Sales Survey published by the LIMRA Secure Retirement Institute. Sales are not reported pre-2012 as the size of the deferred income annuities market is minimal.

annuity market (Panel B). The market expanded rapidly between 2012 and 2014, with the real value of new considerations more than quadrupling in size.



(A) Cumulative Deferred Income Annuities Considerations

(B) Distinct Firms Operating in the Market for Deferred Income Annuities

FIGURE 4. Growth In The Deferred Income Annuities Market

To study the equilibrium relation between price, quantity, and selection I complement this data on aggregate quantities with hand-collected quotes for individual annuity policies sourced from *Annuity Shopper* magazine between 2013 and 2021.[22] The Annuity Shopper magazine was a longstanding product in the annuity and life insurance industry, targeting potential annuitants as well as financial advisors and Human Resources professionals. Consequently, it is in the interests of life insurers to provide quotes on their products.[23]

In each period, national insurance companies provide quotes for a slate of policies. I collect a detailed quarterly panel dataset of the annuity payouts offered by insurers for homogeneous contract terms with 5,882 observations. These policies vary in the payouts

[22]Previously The Annuity Shopper's Buyers Guide. Before the third quarter of 2013 quotes for deferred income annuities are not available and, sadly for retirement researchers, it ceased to operate in 2021.

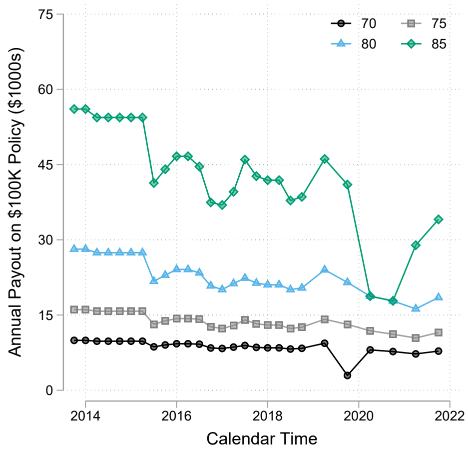
[23]Indeed, over 40 large US insurers provided quotes for a variety of products in the July 2015 issue. Appendix B provides an example testimonial.

they offer, the life insurer who provides the annuity, the age at which the annuity begins to provide an income stream, as well as the current age of the purchaser and their gender. In each period there are up to 28 different quotes available for each insurer varying in their contract terms and I focus on single life non-refundable policies without a guarantee period (Einav et al. 2010, study adverse selection in the guarantee decision). An important advantage of this empirical setting is that the product is homogeneous. This differs health insurance, a common empirical setting, where differences in in- and out-of-network providers lead to differentiated products.

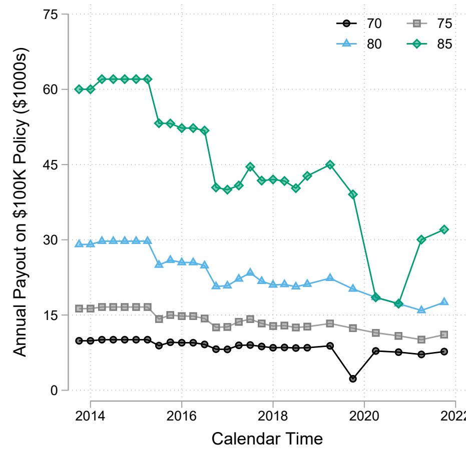
Insurers price annuities for men and women separately, reflecting differences in their survival risk. In addition, annuities are priced separately by the age at which the consumer is purchasing the annuity contract and the the deferral age at which the contract begins paying out. Quotes are available for three purchase ages (45, 55 and 60) as well as 5 different deferral ages (65, 70, 75, 80, and 85). Given policy characteristic at a point in time, quotes provide the annual payout offered per $100,000 paid for the deferred income annuity.

Figure 4B shows how the number of firms offering deferred income annuities changes over time. Broadly, there are two regimes: first, a period where the number of entrants is stable at six distinct insurance firms; and second, a period averaging 10 firms. This expansion in the number of operating firms coincides with reforms to the fiduciary duty of brokers operating in brokerage markets (Egan et al. 2022).

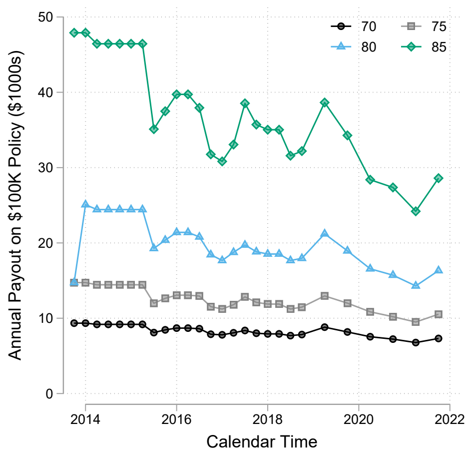
Figure 5 shows payouts from annuities purchased at the age of 65 with policies offered by American General shown in the left column and policies offered by New York Life shown in the right column, two of the largest national insurers. The first row shows policies for men and the second row shows policies for women with each line corresponding to a deferral age from age 70 to 85. Three important facts emerge from this comparison. First, there are clear differences in policy generosity by gender. Second, there are systematic differences by the age at which the contract begins paying



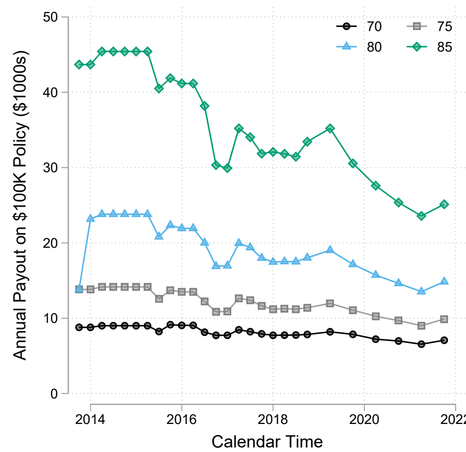
(A) American General (Men)



(B) New York Life (Men)



(C) American General (Women)



(D) New York Life (Women)

FIGURE 5. Average Payouts from Deferred Income Annuity Policies Purchased at Age 65 By Insurer, Gender and Deferral Age

out. Policies with later deferral dates, which expose consumers to greater mortality risk, offer larger payouts on average and systematic differences across men and women are consistent with differences in their expected longevity. This highlights the potential for selection to drive pricing decisions by firms in this market. Finally, comparing across the two columns, there are small deviations in the payout by insurer.

Figure A2 in Appendix B repeats this exercise for contracts offered by New York Life, but conditioning on the deferral age and varying the purchase age. Again, contracts that offer payouts further into the future have higher payout amounts. Despite systematic differences by product characteristics, a notable feature of both Figures 5 and A2 is evidence of a market level trend in payouts over time for all contract terms and for both insurers. This reflects that the largest cost to the insurer is the opportunity cost of foregone investment (see Mitchell et al. 1999) as well as a changing pool of annuitants.

I now turn to measuring selection and maturity, the key quantities of interest, before describing the empirical strategy used to test the implications of the model in Section 2.

3.1. Measuring Selection and Market Maturity

Each policy offered to annuitants at a purchase age, a, of gender, g, is indexed by the deferral age, d, at which payouts begin, the identity of the insurance firm, i, as well as the date, t, at which it is purchased. Each annuity, $\{a, d, g, i, t\}$, offers a fixed income A after the deferral age. I follow Finkelstein and Poterba (2002) and adopt a money's worth calculation to infer selection from annuity payouts. To do this, I calculate the expected present discounted value of the associated annuity income stream

$$EPDV a, d, g, i, t = \frac{1}{1 - \chi_a(g, t-a)} \sum_{h=a}^{110} \frac{A \times \mathbb{1}[h \geq d] \times \chi_h(g, t-a)}{\prod_{j=h-a}^{110-a}(1 + i_j(t))}, \tag{17}$$

where $\chi_h(g, t-a)$ is probability of surviving until age h as a function of annuitant gender and birth cohort $t-a$. The insurer's interest rate over a horizon j from date t is $i_j(t)$. From this I compute the money's worth value of the annuity (its implicit price):

$$MW_{a,d,g,i,t} = \frac{1}{100,000} EPDV_{a,d,g,i,t}. \tag{18}$$

A money's worth value less than one indicates that the annuity is expected to pay out less than the upfront cost and vice-versa. In the absence of annuity product specific mortality tables (see Finkelstein and Poterba 2004 for evidence of differential mortality across products) the money's worth value can be used to infer deviations from the mortality table used to calculate it. In this sense, it allows researchers to infer differential mortality and the presence of adverse selection (see also Warshawsky 1988; Mitchell et al. 1999; Poterba and Solomon 2021).[24] I build a measure of selection using the per dollar deviation from the expected present discounted value under zero adverse selection using population mortality tables.

I use the BBB corporate bond yield at 10-year maturity to determine the insurer's interest rate, i_j, and cohort life tables for the age a population to determine mortality risk $S_h(g)$ at a given point in time (linearly interpolating between successive cohort life tables). The deviation from actuarially fair pricing provides a measure of implicit price and implied adverse selection for each contract. Using gender-specific mortality to calculate money's worth, the implied adverse selection is given by

$$S_{a,d,g,i,t} = 1 - MW_{a,d,g,i,t}, \tag{19}$$

where a larger positive value indicates greater adverse selection.

The testable implications of Section 2 relate adverse selection and market maturity. I proxy unobserved market maturity using the natural logarithm of cumulative sales in the market for deferred income annuities up to that point in time (Figure 4A), $\ln Q_t^{DIA}$. As the measure of new considerations used to construct cumulative sales is not available at a finer disaggregation I use total sales for all deferred income annuities, rather than sales at the characteristic level. This has an intuitive interpretation as a measure of broad market popularity that corresponds to maturity discussed in Section 2. Using a log transformation implies that there is a concave-increasing relationship between maturity and cumulative sales of deferred income annuities and allows for an elasticity

[24] Cannon and Tonks (2016) argue that prudent insurers may deviate from actuarially fair pricing when future mortality risk is uncertain. The empirical strategy below allows for this.

interpretation of the relationship between adverse selection and market maturity.

3.2. Empirical Strategy

Given this measure of adverse selection for each contract and a suitable proxy for market maturity, I now describe the empirical strategy used to test the model predictions. A key threat to identifying the effect on adverse selection is that alternative strategic considerations (e.g., dynamic price discrimination) in maturing markets may generate observationally equivalent pricing patterns even absent adverse selection. Thus the correlation between maturity on pricing may not capture the effect of adverse selection.

Absent private information, it would not be possible for adverse selection to change as the market evolves. Therefore, to address these identification concerns and directly test the model's implications, I use differences in available private information by age to exploit variation in the potential for adverse selection. Specifically, as health inequality increases by purchase age (see, e.g., Hosseini et al. 2022; Russo et al. 2024), private information about the future mortality of potential annuitants is also increasing. Proposition 7 in the general model below formalizes this simple intuition.

This motivates an empirical strategy that exploits differences in the effects of market maturity by consumer's purchase age. I leverage the differential effect of market maturity between those age 45 and those aged 55 and 65. The key identifying assumption is a form of parallel trends assumption: (conditional on other controls) strategic considerations, other pricing decisions, administrative costs, profit rates, and other price-relevant factors would evolve similarly with market maturity if 55 and 65 year olds had the same level of mortality relevant private information as 45 year old consumers. This strategy is implemented by estimating the following specification

$$\begin{aligned}\ln S_{a,d,g,i,t} = \gamma_a + \gamma_d + \gamma_i + \gamma_g + \lambda \ln q_t^{FIA} + \beta \mathbb{1}[a = 45] \times \ln Q_t^{DIA} \\ + \delta \mathbb{1}[a > 45] \times \ln Q_t^{DIA} + u_{a,d,g,i,t},\end{aligned} \tag{20}$$

where γ_a denotes purchase age specific fixed effects, γ_d denotes deferral age specific fixed effects and, γ_g denotes gender specific fixed effects. Collectively, these fixed effects capture systematic differences in selection or administrative costs by contractible plan characteristics that do not vary over time. β captures how pricing evolves with market maturity for age 45 purchasers and δ captures the differential effect for groups with more private information. This is akin to a parametric difference-in-differences estimator as it exploits differences in the within purchase age selection as the market matures. In this analogy, 'treatment' is determined by ex-ante difference in exposure (private information) and estimation uses continuous variation in market maturity.

The parameter of interest is δ which measures the differential effect of market maturity on pricing for groups with more private information (and, hence, a greater propensity for adverse selection). A key testable implication of the model outlined above is that adverse selection declines as markets mature. This corresponds to $\delta < 0$.

As the market matures, other aggregate factors may simultaneously drive differences in the choices of annuitants or the composition of the pool of potential annuitants including population level mortality improvements, differences in the financial resources of retirees over time, or the effect of legislative reform (such as the change to broker fiduciary duty studied by Egan et al. 2022). These factors may be correlated with a positive time trend in market maturity and bias the estimate of maturity on adverse selection. To minimise this threat to identification, I control for contemporaneous demand in other fixed income annuity products excluding deferred income annuity considerations (Figure A1, Appendix B), $\ln q_t^{FIA}$, which absorbs time varying shocks affecting all fixed income annuity products.

Similarly, changes in average prices over time may reflect changes in the composition of insurers due to entry and exit. If insurers serve different populations due to heterogeneous geographic penetration or face different administrative costs, this would confound the effect of market maturity. I control for insurer-specific fixed effects which accounts for compositional changes in firms.

In addition to the specification in equation (20), I also estimate the following:

$$\begin{aligned}\ln S_{a,d,g,i,t} = &\gamma_a + \gamma_d + \gamma_i + \gamma_g + \lambda \ln q_t^{FIA} + \beta \mathbb{1}[a = 45] \times \ln Q_t^{DIA} \\ &+ \delta_{55} \mathbb{1}[a = 55] \times \ln Q_t^{DIA} + \delta_{65} \mathbb{1}[a = 65] \times \ln Q_t^{DIA} + u_{a,d,g,i,t}. \end{aligned} \tag{21}$$

This differs in explicitly examining the heterogeneity between consumers who purchase annuities at age 55 and those who purchase at age 65. Examining this heterogeneity allows for a direct empirical test of a second testable implication, that markets with greater (cost-relevant) private information experience larger reductions in adverse selection as markets mature ($\delta_{65} > \delta_{55} > 0$).

3.3. Adverse Selection Among Early Adopters

Table 1 reports the results from estimating equation (20). The first column only controls for fixed differences across purchase ages. The estimated effect of additional private information on the elasticity of selection (as a deviation from actuarially fair pricing) with respect to market maturity is negative. A 1% increase in past cumulative sales of deferred income annuity products is associated with a –0.191 percentage reduction in selection over and above strategic price dynamics for purchase ages with less private information. Thus, as markets mature adverse selection decreases. This is direct evidence of larger adverse selection among early adopters.

The estimated effect is economically significant and statistically significant at the 1% level; rejecting the null hypothesis of no or advantageous selection. Consistent with the identification argument, I also find that measured selection is on average larger for older purchase ages. Nevertheless, the way in which it *evolves* with maturity drives the estimated effects reported here.

The second column adds additional controls for important contract characteristics:

	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}[a > 45] \ln Q_t$	-0.191^{***}	-0.224^{***}	-0.227^{***}	-0.235^{***}	-0.238^{***}
	(0.034)	(0.030)	(0.028)	(0.029)	(0.027)
FEs					
Gender	*No*	*Yes*	*Yes*	*Yes*	*Yes*
Deferral Age	*No*	*Yes*	*Yes*	*Yes*	*Yes*
Insurer	*No*	*No*	*No*	*Yes*	*Yes*
FIA Demand	*No*	*No*	*Yes*	*No*	*Yes*
N	5, 857	5, 857	5, 857	5, 857	5, 857
R^2	0.110	0.409	0.486	0.440	0.514

Notes: * $p < 0.10$, ** $p < 0.5$, *** $p < 0.01$. Huber-White heteroskedasticity robust standard errors are reported. Estimates reported for coefficient δ in equation (20). Implied selection is recovered using equation (19) and panel data collected from policy quotes printed in *Annuity Shopper* magazine. Aggregate quantities correspond to the cumulative real value of new deferred income annuity considerations collected from LIMRA and new considerations for all other fixed income annuity (FIA) products reported in Figure 4. All Specifications additionally control for purchase age indicators.

TABLE 1. Adverse Selection and Market Maturity

gender and deferral age. As the pricing of annuities varies across these characteristics there is a large increase in the overall explanatory power as measured by the R-squared. However, there is only a small impact on the estimated decline in adverse selection. The elasticity increases, but is overall similar to the previous point estimate. This is because these plan characteristics deliver systematically different pools of annuitants, but evolve similarly over time.[25] The third column adds observed demand for all other fixed income annuities, $\ln q_t^{FIA}$, a proxy for aggregate factors that drive annuity demand and pricing in general. Although this absorbs some of the common effect of market maturity on pricing, it does not affect the point estimate because the empirical strategy exploits differences across purchase ages to identify the component driven by selection.

[25]Estimated effects are qualitatively similar when estimated separately for each gender and deferral age grouping exploiting the same variation in private information across purchase ages. In some cases, these results have less precision (as each subsample is approximately one-tenth of the pooled sample).

The fourth column instead includes insurer specific fixed effects to control for changes in the composition of insurers who may face different pools of annuitants or administrative costs. Overall, the point estimate increases slightly although the differences across columns three and four are statistically insignificant. This reflects the fact that these are national insurers who cover the whole US population and, thus, face similar pools of potential annuitants. Although there are differences in administrative costs across insurers (Mitchell et al. 1999; Koijen and Yogo 2015), these are, on average, only weakly correlated with entry into this market. The final column combines the controls in columns four and five. Across all sets of controls, the estimated elasticity are stable in magnitude and statistically significant at the 1% level. The empirical evidence in this market highlights that adverse selection can be higher among early adopters.

Table 2 reports the results of estimating the specification in equation (21). The key difference is that this disaggregates the elasticity into separate estimates for age 55 and 65 purchasers. As with the results in Table 1, the parameter estimates differ slightly when controls for contract characteristics are added (columns one compared with two). Again, the point estimates are largely stable across alternative controls.

The first row reports the elasticity of selection for age 55 purchasers. These estimates vary between −0.126 and −0.171 and are consistently negative. The second row reports the elasticity of selection for age 65 purchasers. These estimates are larger in magnitude as the pooled effect is the weighted average, varying between −0.273 and −0.323, and are also consistently negative. Like the pooled elasticity they are always statistically significant at the 1% level and reject the null hypothesis of no or advantageous selection in each age group.

Moreover, the disaggregated elasticities allow for a test of the second prediction: all else equal, early adopters are more adversely selected when there is more private information. Estimated elasticities increase with purchase age (and therefore private information) and their difference is negative and statistically significant. This directly

	(1)	(2)	(3)	(4)	(5)
$\mathbb{1}[a = 55] \ln Q_t$	-0.126^{***}	-0.158^{***}	-0.161^{***}	-0.168^{***}	-0.171^{***}
	(0.037)	(0.031)	(0.029)	(0.031)	(0.029)
$\mathbb{1}[a = 65] \ln Q_t$	-0.273^{***}	-0.307^{***}	-0.311^{***}	-0.320^{***}	-0.323^{***}
	(0.035)	(0.030)	(0.028)	(0.029)	(0.027)
FEs					
Gender	*No*	*Yes*	*Yes*	*Yes*	*Yes*
Deferral Age	*No*	*Yes*	*Yes*	*Yes*	*Yes*
Insurer	*No*	*No*	*No*	*Yes*	*Yes*
FIA Demand	*No*	*No*	*Yes*	*No*	*Yes*
N	5, 857	5, 857	5, 857	5, 857	5, 857
R^2	0.115	0.414	0.490	0.445	0.519

Notes: * $p < 0.10$, ** $p < 0.5$, *** $p < 0.01$. Huber-White heteroskedasticity robust standard errors are reported. Estimates reported for coefficient $\delta_5 5$ and $\delta_6 5$ in equation (21). Implied selection is recovered using equation (19) and panel data collected from policy quotes printed in *Annuity Shopper* magazine. Aggregate quantities correspond to the cumulative real value of new deferred income annuity considerations collected from LIMRA and new considerations for all other fixed income annuity (FIA) products reported in Figure 4. All Specifications additionally control for purchase age indicators.

TABLE 2. Adverse Selection and Market Maturity by private information

tests the second testable implication and also lends support to the identifying assumption of less private information for younger age groups.

Robustness and Alternative Mechanisms. The empirical evidence presented here uses differences in the availability of private information by purchase age to identify the dynamics of adverse selection in a maturing market and is consistent with testable implications from theoretical analysis above. In Appendix B I provide further results and probe into the robustness of these results. In particular, I show that these results are robust to alternative controls for common aggregate factors as well as a more granular set of controls for policy characteristics. Moreover, the qualitative insights are robust to alternative specifications of the dependent variable including selection in levels and calculating implied selection using alternative mortality tables.

Nevertheless, it may be the case that alternative mechanisms produce these heterogeneous dynamics by purchase age. I briefly discuss two potential mechanisms and the reasons that they are unlikely to be responsible for the dynamics in this market.

First, in a new market firms may be learning about the distribution of risks (cost types) and may do so differentially among the pools of potential annuitants at different purchase ages. This could lead to initial miss-pricing and induce a negative correlation. However, although deferred income annuities are a new product, insurers are experienced in pricing annuity and life-insurance contracts based on the distribution of underlying mortality risk. As equation (19) shows, the actuarial formula for a deferred income annuity payout is almost identical to the formulas for these existing products. They differ only in the timing at which payout begins. Therefore, it would have to be the case that insurers were learning about the declining adverse selection by age as the market matures – precisely the model's prediction.

Second, firms may initially 'invest' in market power, e.g. by undercutting or offering 'sweetheart' deals, and subsequently raise markups to 'harvest' downward sloping residual demand.[26] This would produce a negative contemporaneous correlation between equilibrium prices and demand, a distinct implication to the time path considered here. Additionally, this mechanism has sharp predictions about the declining number of insurers in the market which is at odds with Figure 4. The number of insurers offering these contracts grows over time – inconsistent with price and quantity dynamics driven by this form of strategic behaviour.

[26]This may arise if consumers do not switch products due to search costs. This is unlikely to be of relevance here where the annuitization decision is typically once and for all.

4. Generalized Problem

The implications derived in Section 2 rely on monotonicity. As Fang and Wu (2018) note, selection problems with multidimensional types do not generically satisfy monotonicity.[27] I now provide sufficient conditions for monotonic cost curves and adverse selection under arbitrary marginals – a prerequesite for unraveling. Subsequently, I formalize how characteristics of immature and mature markets affect this unraveling.

4.1. Sufficient Conditions for Monotonicity and Adverse Selection

With multidimensional heterogeneity, the critical assumption for adverse selection is this dependence between individual willingness-to-pay and costs (See Fang and Wu 2018). In general, this depends on the shape of the copula which represents their joint distribution.[28] Copulas encode the correlational structure between π_i, r_i, and ε_i without imposing a specific parametric family of distributions and decouple assumptions on dependence properties from those on marginal distributions.[29] They encode this structure as restrictions on the shape of rank dependence or sorting in quantile space (Anderson and Smith 2024). Using copulas allows for the role of *sorting* to be disentangled from the distribution of costs and risk premia.

[27] This motivates Fang and Wu to consider local adverse (and advantageous) selection in a multidimensional context. Their Definition 2 corresponds to condition (i) in Proposition 4 below and the global conditions have a natural analogue for monotone regions and local adverse selection.

[28] Copulas have many uses in economics and finance – particularly in the context of selection and dependent risks. Recent examples include competing hazard models (Kim 2021), and multi-dimensional skills (Gola 2021). The main representation result is Sklar's theorem, which I reproduce below.

Sklar's Theorem: Let F be a joint distribution function for a d-dimensional random vector $(X_1, X_2, \ldots, X_d)$, and let F_i be the marginal distribution function for the i-th component X_i. Then, there exists a copula C such that for any $x_1, x_2, \ldots, x_d$ in the real line:

$$F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F_2(x_2), \ldots, F_d(x_d))$$

Moreover, if X is continuous, then the Copula C is unique.

[29] Copulas are invariant to rank-preserving transformations. Therefore, results immediately apply to multiplicatively separable utility, $u = exp(\cdot)$, and when insurer costs are transformed cost types.

The introduction of copulas is more than a technical abstraction. Although results can be characterized for the special case of joint-normality, this comes at the cost of restricting the risks that consumers experience and insurers insure. In addition, normality rules out heavy-tails and skew in the distribution of risks which are an important concern in actuarial modeling because tail events have large impacts on the balance sheets of insurers and financial institutions despite their infrequency. Consider the following example, motivated by the empirical specification in Cohen and Einav (2007), for an illustration of the limits of normality: consumers have constant absolute risk aversion given by θ, and face a financial loss m which arrives with probability λ. Then we have the following expressions for the certainty equivalent, expected cost, and risk premia

$$\begin{aligned} &\text{Certainty Equivalent:} && CE_i = \frac{\ln(1-\lambda_i+\lambda_i e^{-\theta_i m})}{\theta_i} \\ &\text{Expected Cost:} && \pi_i = \lambda_i m \\ &\text{Risk Premia:} && r_i = \frac{\ln(1-\lambda_i+\lambda_i e^{-\theta_i m})}{\theta_i} - \lambda_i m \,. \end{aligned}$$

This example incorporates the type of rich, multidimensional heterogeneity that has been the focus of this paper. When risk aversion and the the arrival rate vary across individuals the distribution of risk premia is non-normal even when λ and θ are (log-)normally distributed. Thus, although the language of copulas increases the complexity of the results it also allows the general framework to describe economically and empirically relevant settings such as the simple insurance contract above.

The following proposition provides sufficient conditions on the joint-distributions for monotonicity without placing any restrictions on the marginal distributions. Alternatively, these can be viewed as the precise assumptions for imposing shape restrictions in non-parametric estimation.

Proposition 4 (Adversely Selected Markets). *Let copula $\hat{C}$ denote the dependence between cost type, π_i, and willingness-to-pay, ω_i. Then*

i. *[Component-wise Concavity] The market is adversely selected, $MC(p)$ is increasing in price, if $\hat{C}(u, v)$ is concave in v for all $u \in (0, 1)$.*

ii. *[Corner Set Montonicity] Average costs, $AC(p)$, are monotonically increasing in price if $(u-\hat{C}(u,1-v))/v$ is increasing in v for all $u \in (0, 1)$.*

The formal proof is given in Appendix C. To maximise accessibility, I present the proofs for the special case of joint-normality in the main text. Suppose π and $\tilde{r}$ are multivariate normal with correlation $\rho_{\pi,\tilde{r}}$. Then the joint (normal) distribution of cost type and willingness-to-pay is given by,

$$\begin{pmatrix} \pi_i \\ \omega_i \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_\pi \\ \mu_\omega \end{pmatrix}, \begin{pmatrix} \sigma_\pi^2 & \rho\sigma_\pi\sigma_\omega \\ \rho\sigma_\pi\sigma_\omega & \sigma_\omega^2 \end{pmatrix}\right],$$

where

$$\rho = \frac{\sigma_\pi + \rho_{\pi,\tilde{r}}\sigma_{\tilde{r}}}{\sigma_\omega} > 0 \,. \tag{22}$$

Consequently, the marginal cost curve has the following closed form expression:

$$MC(p) = E(\pi|\omega = p) = \mu_\pi + \rho\frac{\sigma_\pi}{\sigma_\omega}(p - \mu_\omega), \tag{23}$$

which is increasing in p when $\rho_{\pi,\tilde{r}}$ (and thus ρ) is positive. In this case, average costs are the expected value of a truncated normal distribution. The average cost curve is

$$AC(p) = E(\pi|\omega \geq p) = \mu_\pi + \rho\sigma_\pi\frac{\phi(\frac{p-\mu_\omega}{\sigma_\omega})}{1 - \Phi(\frac{p-\mu_\omega}{\sigma_\omega})}, \tag{24}$$

which again increase in p for positive $\rho_{\pi,\tilde{r}}$.

More formally, conditions (i) and (ii) restrict the (scale invariant) rank dependence *within* markets and formalize the intuitive notion that larger values of X tend to go with larger values of Y. It implies positive quadrant dependence and, therefore, both positive values of standard correlation and linear regression coefficients

Note, for the normal distribution, positive correlation induces adverse selection.[30] This is not true in general. Proposition 4 presents alternative conditions that ensure adverse selection and the monotonicity of marginal costs. Another sufficient condition is the familiar monotone likelihood ratio property (Milgrom 1981).[31]

LEMMA 1 (MLRP). *Let the density $f()$ satisfy the Monotone Likelihood Ratio Property, i.e. $f(\omega_1|\pi_1)/f(\omega_1|\pi_2) \geq f(\omega_2|\pi_1)/f(\omega_2|\pi_2)$ for any $\omega_1 \geq \omega_2$ and $\pi_1 \geq \pi_2$, then conditions (i) and (ii) in Proposition 4 hold.*

Intuitively, the MLRP property allows the insurer to infer higher costs the signal of higher willingness-to-pay. Corollary A1 in Appendix C provides a reinterpretation of the MLRP conditions in terms of the joint distribution of π and $\tilde{r}$.[32]

I now provide assumptions on the primitives of insurance markets in immature and mature markets captured by the joint distribution of π_i and $\tilde{r}_i$ which is assumed to satisfy the conditions in Proposition 4. These assumptions allow me to draw precise predictions about supply and demand as well as equilibrium prices, quantities and consumer surplus as well as to provide an ordering over these equilibrium outcomes for

[30]For parametric copula families these conditions restrict the parameter space, for example requiring the correlation coefficient is positive for the normal copula. This relates to the intuitive misconception that positive correlation is sufficient for adverse selection, however, this is in fact a consequence of the conditions in Proposition 4. Furthermore, Appendix C shows how the joint distribution of ω and π can be derived from assumptions on the joint distribution of π and $\tilde{r}$.

[31]A weaker necessary condition is a log-concave survival function, often invoked in studies of incomplete information (Milgrom and Weber 1982, Chade and Schlee 2012, Chade and Swinkels 2021).

[32]This shows that the MLRP condition depends on the copula density function (given by the cross partial derivative of the copula C). The examples that follow shows that because dependence is measured in ranks, the cross partial derivative captures more than the correlation between π and $\tilde{r}$. It additionally encodes information about how jointly heavy-tailed π and $\tilde{r}$ are. When the copula and marginals are gaussian the correlation coefficient is a sufficient statistic to summarize the cross-partial derivative.

markets with common distributions of π_i. Thus, while I motivated the assumptions in the context of studying market maturity, the comparative statics are widely applicable, e.g. when markets are indexed by time or space. Equally, similar assumptions allow for the ranking of advantageously selected markets

4.2. Differences between Mature and Immature Markets

I use superscripts *MM* to denote the mature insurance market and *IM* to denote the immature insurance market for a new form of innovative insurance coverage.

The key assumption in Section 2.2 is that willingness-to-pay is depressed in immature markets relative to mature markets – lowering demand. This relative lack of demand can come from objective differences in risk premia, r, or differences in demand frictions, ε. I formalize this in Assumption 1 which defines cases where the marginal distribution of risk premia net of frictions are ordered in a stochastic sense, where $X \preceq Y$ denotes the relation Y first order stochastic dominates X.

ASSUMPTION 1. *The perceived risk premia (net of any behavioral influences) of insurance contracts are lower for immature markets.* $\tilde{r}^{MM} \succeq \tilde{r}^{IM}$

This formalizes shifts in the risk premia and demand frictions discussed above. Instead of demand frictions shifting systematically between immature and mature markets, it may be the case that consumers beliefs about new products are noisier. This would be reflected in a greater dispersion of demand frictions in immature markets, which captures unbiased, but more diffuse, priors over the value of the insurance contract. Here, if the distribution of objective risk premia also moves between mature and immature markets, Assumption 1 is weaker than requiring stochastic dominance of objective risk premia, r, or demand frictions, ε. It only needs to be satisfied *on net.*

In contrast to Spinnewijn (2017), Assumption 1 does not require consumers are, on average, unbiased. Systematic optimism or pessimism in mature markets is possible.

Even in established selection markets, such as annuities or insurance, there is substantial evidence of choice frictions (Brown et al. 2021), biased beliefs (O'Dea and Sturrock 2023), probability distortions (Barseghyan et al. 2013), and limited consideration (Barseghyan et al. 2021).

Assumption (2) formalizes that these are mature and immature markets for an identical insurance product with identical potential and realized risk pools.

Assumption 2. *Risk types, π_i, have identical marginal distributions in mature and immature markets.*

These assumptions restrict how marginal distributions differ between mature and immature markets. Yet, they do not capture how the sorting of consumers into insurance may change as markets mature. These differences in sorting can occur because of changes in consumers' self insurance behaviour or behavioural frictions. This sorting can improve the match of individuals to insurance plans, but also change how purchasing insurance signals an individual's cost. The following restriction on rank dependence capture differences in sorting patterns, generalizing the correlation.

Assumption 3. *Resorting satisfies the following two conditions for the copulas C, the bivariate copula denoting the dependence between π_i and $\tilde{r}_i$, and $\hat{C}$:*

i. The partial derivatives of C satisfy $\frac{\partial C^{MM}(u, F_{\tilde{r}}^{MM}(\tilde{r}))}{\partial u} \leq \frac{\partial C^{IM}(u, F_{\tilde{r}}^{IM}(\tilde{r}))}{\partial u} \quad \forall\, u \in [0,1]$ *and* $\tilde{r}$.

ii. The copulas $\hat{C}$ satisfy the following concordance ordering (Yanagimoto and Okamoto 1969; Tchen 1980; Scarsini 1984): $\quad \hat{C}^{IM}(u,v) \geq \hat{C}^{MM}(u,v)\ \forall\, u, v \in [0,1]$

This orders the joint distribution *between* markets by their implied dependence. First, a higher rank along the cost dimension of a consumer's type is associated with a larger change in their joint probability in immature markets. This is a technical assumption required to rule out demand expansions in immature markets. The second restriction establishes costs and willingness-to-pay are larger in concordance. Formally, their joint density is more tightly concentrated around a monotone graph.[33] Thus, the sorting on cost has increased. This second part of the assumption will be key to derive the implications below.[34] This is because increased sorting on cost changes the extent to which the decision to purchase insurance provides a signal to the insurer. This then changes insurers' inference about the latent cost of the consumer at any given price. Intuitively, this restricts negative re-sorting between cost and risk premia that offsets the shift in risk premia. In economic terms, it also rules out self insurance patterns that reverse risk exposure between mature and immature markets. This rules out settings in which early adopters are more risk averse and, consequently, more advantageously selected (see Cutler et al. 2008 and Fang et al. 2008 for empirical evidence on advantageous selection)—although analogous comparative statics apply for this case.

Identical copulas, which imply no re-sorting effects, trivially satisfy Assumption 3. For common parametric copula families, Assumption 3 is a restriction on parameter values, e.g. the correlation ($\rho^{IM}_{\pi,\tilde{r}} \geq \rho^{MM}_{\pi,\tilde{r}}$) for normal copulas. More formally: larger quantiles of risk premia net of frictions, $\tilde{r}$, are associated with a larger increase in the cumulative density of costs in immature markets and vice-versa.

While results for the general model rely only on the above assumptions, closed form solutions are available for linear utility, $u(x) = x$, and when copulas and marginals are normally distributed – which satisfy the conditions in Proposition 4 and Assumption 3. As discussed above, this increases transparency, but imposes strong additional restrictions, including allowing consumers to impose negative costs on the

[33]Both Anderson and Smith (2024) and Boerma et al. (2023) study concordance orderings as the outcome of assignment problems. Anderson and Smith (2024) show how the concordance order (which is identical to their positive quadrant dependence order with constant marginals) increases as the local synergies of the output function increase. Boerma et al. (2023) show that optimal assignments are larger in concordance when mismatch costs become less concave.

[34]It is tempting to conclude that the first property implies the second. This would be true if the assumption were strengthened to $\frac{\partial C^{MM}(u,v)}{\partial u} \leq \frac{\partial C^{IM}(u,v)}{\partial u} \quad \forall\, u, v \in [0, 1]$. However, this implies that conditional densities over ranks satisfy a first order stochastic dominance and, thus, the mean of the rank distribution is larger in immature markets. If this were true, at least one of C^{MM} and C^{IM} cannot satisfy the definition of a copula.

insurer and limiting tail risk. I provide proofs in the text under the following assumption to sketch the intuition behind the general case provide proofs for the general non-parametric case in Appendix D.[35]

ASSUMPTION 4 (Joint Normality). *π_i and $\tilde{r}_i$ are jointly normally distributed. Additionally, the variance of willingness-to-pay is constant, $\sigma_\omega^{IM} = \sigma_\omega^{MM}$, and correlation between risk type and willingness-to-pay is weaker in mature markets, $corr(\pi, \omega^{IM}) \geq corr(\pi, \omega^{MM}) > 0$.*

4.3. Equilibrium in Mature and Immature Markets

I now turn to understanding the implication of Assumptions (1) - (3) on quantities, costs, and the market equilibrium.

First, at any given price, the average individual has a larger willingness-to-pay in mature markets and consequently more individuals choose to purchase insurance: a *level effect* on demand (Panel A of Figure 2) formalized by Proposition 5.

PROPOSITION 5 (Demand Contraction). *Willingness-to-pay is on average larger and demand expands in mature markets. $D^{IM}(p) \leq D^{MM}(p) \quad \forall p$*

PROOF. As the sum of the normal variables is itself normal, the demand curve is

$$D(p) = 1 - F_\omega(p) = 1 - \Phi\left(\frac{p - \mu_\omega}{\sigma_\omega}\right). \tag{25}$$

Assumptions (1) - (2) and (4) imply $\mu_\omega^{IM} \leq \mu_\omega^{MM}$ and σ_ω is constant, then distributions of ω for mature markets first order stochastic dominate those of immature markets, $F_\omega^{MM}(p) \leq F_\omega^{IM}(p)$, which delivers the result. □

[35]To simplify exposition Assumption 4 rules out changes in the dispersion of willingness-to-pay across mature and immature markets. Under this condition, changes to the primitives take the form of location shifts for the marginal distributions. This is weaker than the stochastic ordering of normals as it does not impose a common variance across mature and immature markets for π_i and $\tilde{r}_i$.

Constant dispersion in willingness-to-pay is required for the normal distribution as the support is unbounded. Without this, equivalent results are available with truncated tails (Levy 1982).

This level effect does not change the identity of marginal (and inframarginal) consumers at any quantity demanded is the same and the equilibrium price increases and demand falls (See Proposition 2).

Moreover, when there is *only* a level effect the following lemma shows that the utility function can be generalized to allow for arbitrary substitution patterns between π_i.

LEMMA 2 (Arbitrary Substitutes). *When there is no re-sorting ($C^{IM} = C^{MM}$), Proposition 5 can be extended to any willingness-to-pay satisfying*

$$\omega_i = u(\pi_i, \tilde{r}_i) \tag{26}$$

where $u(\cdot, \cdot)$ is a function increasing in its arguments.

The proof is given in Appendix D. For the special case of the normal distribution, demand contractions are tightly linked to the price elasticity of consumers in immature and mature markets. The following corollary formalises this for the general case:

COROLLARY 2 (Price Elasticity of Demand). *When willingness-to-pay is smaller in the hazard rate order in mature markets, that is*

$$\frac{f^{IM}(\omega)}{1 - F^{IM}(\omega)} \geq \frac{f^{MM}(\omega)}{1 - F^{MM}(\omega)} \; \forall p, \tag{27}$$

then demand is more price sensitive in immature markets.

PROOF. This follows from substituting the definition of demand into the usual formula for the elasticity,

$$\eta_{D,p} = D^{'}(p)\frac{p}{D(p)} = -f(p)\frac{p}{1 - F(p)}, \tag{28}$$

which implies

$$\eta^{IM}_{D,p} - \eta^{MM}_{D,p} \propto - \underbrace{\left(\frac{\phi(\frac{p-\mu^{IM}_{\omega}}{\sigma_{\omega}})}{1-\Phi(\frac{p-\mu^{IM}_{\omega}}{\sigma_{\omega}})} - \frac{\phi(\frac{p-\mu^{MM}_{\omega}}{\sigma_{\omega}})}{1-\Phi(\frac{p-\mu^{MM}_{\omega}}{\sigma_{\omega}})} \right)}_{\text{level effect}\geq 0} \leq 0 \tag{29}$$

The inverse Mills ratio and the difference in the inverse Mills ratio are always positive by the log concavity of the normal distribution ensuring non-negativity. Thus consumers are more price elastic in immature markets. □

Note that this is always true under joint-normality and, unlike the usual stochastic order used elsewhere in this paper, it is not in general preserved under convolution.[36] Corollary 2 therefore provides stronger assumption on demand that guarantees this in the general case. As with the discussion in Section 2.2, I now turn to the role of sorting.

PROPOSITION 6 (Increased Adverse Selection). *Consumers are adversely selected in both mature and immature markets. If resorting between markets satisfies the following additional assumption:*

i. [Monotone Regression Dependence Ordering] $\tilde{g}^{IM}_u(g^{MM}_u(v))$ is increasing in v for all $u \in (0,1)$, where $g_x(y) = \partial\hat{C}(x,y)/\partial y$ and $\tilde{g_x} \equiv g_x^{-1}$,

Then consumers are more adversely selected in immature markets $\left(\frac{\partial MC^{IM}(p)}{\partial p} \geq \frac{\partial MC^{MM}(p)}{\partial p} > 0\right)$

PROOF. Consider the joint (normal) distribution of cost type and willingness-to-pay,

$$\begin{pmatrix} \pi_i \\ \omega_i \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_\pi \\ \mu_\omega \end{pmatrix}, \begin{pmatrix} \sigma^2_\pi & \rho\sigma_\pi\sigma_\omega \\ \rho\sigma_\pi\sigma_\omega & \sigma^2_\omega \end{pmatrix} \right],$$

[36] See Theorem 1.B.4 in Shaked and Shanthikumar (2007) for a special case.

where

$$\rho = \frac{\sigma_\pi^2 + \rho_{\pi,r}\sigma_\pi\sigma_r}{\sigma_\omega\sigma_\pi} > 0 \; . \tag{30}$$

Then the marginal cost curve has the following closed form expression:

$$MC(p) = E(\pi|\omega = p) = \mu_\pi + \rho\frac{\sigma_\pi}{\sigma_\omega}(p - \mu_\omega), \tag{31}$$

which is increasing in p when ρ is positive. The second part of the proposition then follows from Assumption 4. □

Thus marginal cost curves are steeper. The proof in the general case follows from a direct application of Proposition 2.1 in Fang and Joe (1992).[37]

Unfortunately, it is difficult to express the additional assumption required in the general case as lower level restrictions on the joint distribution between cost type and risk premia (see Appendix C.1 for a derivation of the copula $\widehat{C}$ using C). While this assumption lacks intuitive appeal, however, it is only required for this specific result. This is because the usual definition of adverse selection invokes the slope of the marginal cost curve. While this is useful to distinguish adverse and advantageous selection, the notion of being *more* selected it gives rise to is too stringent. Instead, the characterisation of the equilibrium in this paper use results on how adverse selection determines average and marginal costs *in levels* and for a *given demand*. These are derived under the milder assumptions presented thus far.

Further comparing the marginal cost curves in mature and immature markets gives the following lemma relating changes in adverse selection to changes in sorting.

LEMMA 3 (Adverse Selection In Levels). *When there is no re-sorting* ($\widehat{C} = \widehat{C}^{IM} = \widehat{C}^{MM}$)

[37] Again, for parametric families of copulas satisfying this assumption is typically a restriction on the parameter space. See Joe (2014) Chapter 4 for a systematic review.

marginal costs are higher at any price. However, when re-sorting occurs, effects on marginal cost are ambiguous.

While the slope of the marginal cost curve increases, the effect of re-sorting has an ambiguous effect on the level of marginal costs depending on the price.

I make this formal using expression (31) for the marginal cost in the normal case:[38]

COROLLARY 3 (Decomposing Marginal Costs). *Under Assumption 4, the differences in marginal costs between mature and immature markets can be decomposed into the level effect of demand and a re-sorting effect*

$$MC^{IM}(p) - MC^{MM}(p) = \frac{\sigma_\pi}{\sigma_\omega}\rho^{IM}\left[\underbrace{(\mu_\omega^{MM} - \mu_\omega^{IM})}_{\text{level effect}\geq 0} + \underbrace{\left(1 - \frac{\rho^{MM}}{\rho^{IM}}\right)(p - \mu_\omega^{MM})}_{\text{re-sorting effect}}\right]. \tag{32}$$

This result makes the intuition for the preceding lemma explicit. Absent re-sorting, the increase in marginal costs depends only on the contraction in demand as the second term is equal to zero. However, when there is re-sorting the sign of the second term is ambiguous. For prices above the average willingness-to-pay in mature markets, re-sorting increases the difference in marginal costs between mature and immature markets. When prices are lower, however, the effect is negative as those purchasing insurance at high prices in immature markets are more adversely selected. Consequently, the pool of remaining consumers at low prices have lower costs.

Nevertheless, Proposition 7 highlights that, despite ambiguity at the margin, when consumers are reordered and selection on cost increases, then insurers face higher *average* costs.

[38] It is always possible to define a cost curve holding sorting fixed, $MC^{MM,nosort}$, and decompose the general case as $\underbrace{\left(MC^{IM}(p) - MC^{MM,nosort}(p)\right)}_{\text{level effect}\geq 0} + \underbrace{\left(MC^{MM,nosort}(p) - MC^{MM}(p)\right)}_{\text{re-sorting effect}}$.

PROPOSITION 7 (Cost Expansion). *Average costs are increasing in price and early adopters are higher cost on average, $AC^{IM}(p) \geq AC^{MM}(p)$.*

PROOF. Under joint normality, average costs are the expected value of a truncated normal distribution. The average cost curve is

$$AC(p) = E(\pi|\omega \geq p) = \mu_\pi + \rho\sigma_\pi \frac{\phi(\frac{p-\mu_\omega}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega}{\sigma_\omega})}. \tag{33}$$

The average cost depends on the inverse Mills ratio of the subjective willingness-to-pay, which captures the extent to which selection truncates the distribution of cost types.

Comparing across markets gives $\forall\, p$

$$\frac{AC^{IM}(p) - AC^{MM}(p)}{\sigma_\pi \rho^{IM}} = \underbrace{\left(\frac{\phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})} - \frac{\phi(\frac{p-\mu_\omega^{MM}}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega^{MM}}{\sigma_\omega})} \right)}_{\text{level effect}\geq 0} + \underbrace{\left(1 - \frac{\rho^{MM}}{\rho^{IM}}\right) \frac{\phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})}}_{\text{re-sorting effect}\geq 0} \geq 0 \tag{34}$$

The inverse Mills ratio and the difference in the inverse Mills ratio are always positive by the log concavity of the normal distribution ensuring non-negativity. □

Again this expression can be decomposed into the contribution from the level and re-sorting effects. In contrast to marginal costs, both effects work in the same direction (raising average costs at a given price), as additional selection on costs among inframarginal consumers dominates the ambiguous effect on the marginal consumer.

The first term in (34) captures changes in the level of demand, holding the consumer sorting constant, on the costs of early adopters. The second term in (34), which captures re-sorting, holds fixed the level of selection in immature markets and accounts for the additional sorting on costs which occurs.

The size of both effects are increasing in the share of heterogeneity in willingness-to-pay coming from cost type, π, while resorting effects depend on the amount of selection in immature markets. When this selection is large, insurance purchase is a stronger signal of cost. Consequently, increased sorting magnifies existing selection.

Increased sorting on cost in immature markets, relative to mature markets, results in purchase of the insurance signaling higher costs in the immature market. As sorting on cost increases, consumers with the highest willingness-to-pay have the highest costs leading to higher marginal costs at low quantities and the remaining pool of consumers at high quantities have lower marginal costs. Proposition 8 formalizes this intuition.

PROPOSITION 8 (Cost Expansion Fixing Demand). *Average Costs as a function of quantity are larger in immature markets, $AC^{IM}(q) \geq AC^{MM}(q)$. Marginal Costs rotate around a point and are higher at low quantities, but smaller at high quantities, thus $MC^{IM}(q) \geq MC^{MM}(q)$ for all $q \leq \tilde{q}$ and vice versa. Absent re-sorting, both cost curves are identical.*

PROOF. Substituting inverse demand into (33) gives average costs as a function of the quantity demanded,

$$AC(q) \equiv E\left(\pi | \omega \geq D^{-1}(q)\right) = \mu_\pi + \rho\sigma_\pi \frac{\phi(\Phi^{-1}(1-q))}{1-\Phi(\Phi^{-1}(1-q))} = \mu_\pi + \rho\sigma_\pi \frac{\phi(\Phi^{-1}(1-q))}{q}. \quad (35)$$

Differences in average costs are then

$$AC^{IM}(q) - AC^{MM}(q) = \rho^{IM}\sigma_\pi \underbrace{\left(1 - \frac{\rho^{MM}}{\rho^{IM}}\right) \frac{\phi(\Phi^{-1}(1-q))}{q}}_{\text{re-sorting effect}} \geq 0 \;\; \forall q. \quad (36)$$

Equivalently, substituting inverse demand into (23) gives marginal costs as a function of quantity,

$$MC(q) \equiv E\left(\pi | \omega = D^{-1}(q)\right) = \mu_\pi + \rho\sigma_\pi\Phi^{-1}(1-q). \quad (37)$$

It follows that

$$MC^{IM}(q) - MC^{MM}(q) = \underbrace{(\rho^{IM} - \rho^{MM})(\sigma_\pi \Phi^{-1}(1-q))}_{\text{re-sorting effect}}$$

which shows that the marginal costs are higher at any quantity demanded below the median. This is because the final term is the quantile function of the standard normal distribution (evaluated at $1-q$), thus it is negative when $1-q < 1/2$. This corresponds to the rotation of marginal costs around the median. □

Panel B of Figure 2 above shows the implication of this outward expansion of the average cost curve in immature markets. The remaining results in Section 2.2 rely only on the monotonic and the resulting shifts and rotations in demand and supply. This section establishes those prerequisites and, thus, the remaining results in Section 2.2 follow directly for the general case.

5. Discussion

This paper shows the existence of a novel dynamic free-rider problem that can arise in adversely selected markets. Adverse selection may be more severe in a newly emerging market if demand for the new product is lower in the short run or if consumers sort more strongly on cost. This leads to early adopters being more concentrated among higher cost individuals. This can make new markets unlikely to mature because it generates a new type of free-rider problem. Initial entrants pay the costs of the initial adverse selection, but subsequent entrants compete away long-run rents. This prevents entrants from using potential long-run rents to cross-subsidize initial losses and, in equilibrium, the market never exists. Consequently, this initial adverse selection can result in no entry, and unraveling, even when the mature market is efficient. Counter-intuitively, I show that in this context market power actually improves market outcomes by allowing a monopolist to cross-subsidize early losses with longer-run gains.

This highlights a novel free-rider problem and shows that, when market primitives are dynamically evolving, the usual interplay between adverse selection and market power is reversed. I show the existence of this problem in a simple model of market entry. I then show that period profits are microfounded by the equilibrium quantity and pricing in a standard model of adverse selection with multi-dimensional heterogeneity. Under economically motivated and plausible assumptions on risk premia and the potential biases of consumers, early adopters are more adversely selected and impose higher costs on insurers even though the distribution of potential costs to the insurer are constant. This increases under-insurance and lowers welfare in immature markets.

I provide intuitive results for the monotone case and also derive general comparative static results for the equilibrium of selection markets under direct assumptions on the joint density of model primitives. This develops a partial ordering of markets with multi-dimensional heterogeneity. Markets are ordered by the primitives of their economic environment and I show how this delivers an ordering over equilibrium prices, quantity, selection, and welfare that can be represented using the popular graphical framework for insurance analysis (Einav and Finkelstein 2011).

I use these results to microfound missing markets, where firms do not offer potentially valuable insurance contracts. This extends the classic study of market unraveling in insurance markets to consider this dynamic form of market failure. This jointly explains low utilization, slow demand for new products, and market inactivity.

While it is not possible to study missing markets, I show that the predictions of this theoretical exercise are consistent with observed equilibrium prices and quantities in an emerging market for insurance – the deferred income annuity. I combine data on market size, contract terms, and insurer cost to show that, in equilibrium, selection declines as the market grows using an identification strategy that differentiates this from other price dynamics.

While I apply these tools to the study of insurance markets, this analysis applies more generally for a class of selection markets – where consumers can impose heterogeneous costs on the seller when they choose to purchase – under multidimensional heterogeneity. Crucially, th implications do not depend on whether market immaturity affects the true welfare value of insurance or only consumer's perception of the value. It thus encompasses a variety of applications in economics. These same tools can be used to study markets across time and place. For example, to understand why shifts in risk premia lower adverse selection and derive sharp predictions on the lemons penalty as in Blundell et al. (2019).

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Online Appendix

Appendix A. Proofs for Section 1

Proof of the first part. In immature markets let $-C < 0$ denote the loss when both firms enter and $\Pi_M^{IM} < 0$ denote the monopolist profit, respectively. Equivalently Π_M^{MM} denotes the monopolist profit and $\Pi^{MM} = 0$ denotes the (competitive) profit in mature markets.

	enter	exit
enter	(0, 0)	(Π_M^{MM},0)
exit	(0, Π_M^{MM})	(0,0)

(a) Mature Market

	enter	exit
enter	(-C, -C)	(Π_M^{IM},0)
exit	(0, Π_M^{IM})	(0,0)

(b) Immature Market

TABLE A1. Second Period Payoff Matrix

Payoffs in the second subgame when at least one firm entered in the first period are given in Panel A of Table A1.

In a mature market that does not fully unravel $\Pi_M^{MM} > 0$ and entering is a profitable deviation for both firms if neither enter, thus this cannot be a nash equilibrium of the subgame. There is no profitable deviation from the symmetric strategy profile (enter, enter) as each firm also earns zero under unilateral deviation.

Next, consider the second subgame when neither firm entered in the first period (Panel B). When $\Pi_M^{IM} < 0$, exiting is a strictly dominant strategy and there is a unique equilibrium.

Taking these contingent strategy profiles as given, the normal form representation of the first subgame is in Table A2. As the market is immature and firms earn zero equilibrium profits in mature markets, these payoffs are identical to the immature market second period subgame and exit is strictly dominant.

	enter	exit
enter	$(-C+0, -C+0)$	$(\Pi_M^{IM}+0, 0+0)$
exit	$(0+0, \Pi_M^{IM}+0)$	$(0+0, 0+0)$

TABLE A2. First Period

Therefore, the following symmetric strategy profile is an SPE:

- First period: Play *exit*
- Second period: Play *exit* if no firm has entered, otherwise *enter*

and in equilibrium no firm enters in either period.

Proof of second part. When the market is mature in the second period, the SPE has firms playing strictly dominant strategies. Therefore, only refinements for the immature second period subgame need to be considered.

As this is a two player, two action game, the set of trembling hand perfect equilibria is equivalent to the set of equilibria in which no player plays a weakly dominated strategy. The equilibria considered above (exit, exit) satisfies this. However, the asymmetric strategy profiles in the second round, either (entry, exit) or (exit, entry), do not because entering is weakly dominated by exiting.

This leaves a unique equilibria surviving the trembling-hand refinement in this subgame. The same argument can be applied in the same fashion to the first period delivering a unique equilibria of the extensive form game.

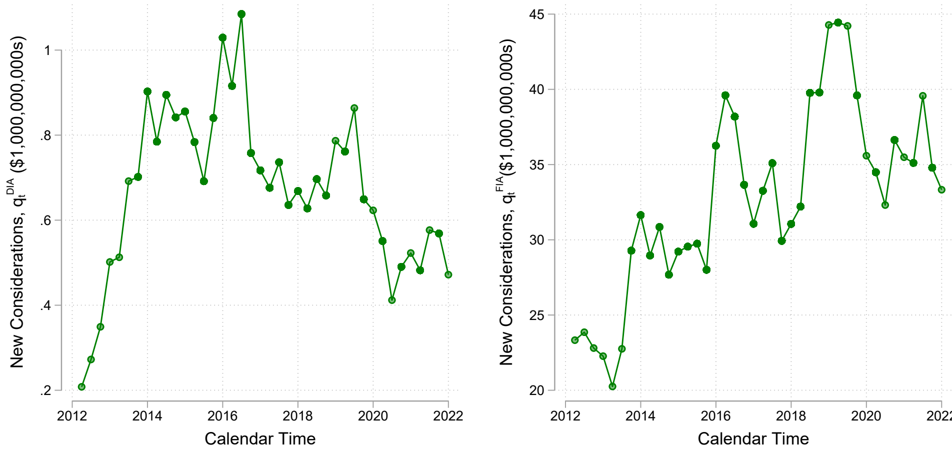
Appendix B. Additional Discussion of Section 3

Testimonials. To highlight the widespread use of the Annuity Shopper quotes I reproduced the following testimonial from the July 205 issue. The testimonial is addressed to Hersh Stein (the publisher):

> Dear Hersh:
>
> Thank you for providing the WebAnnuties packet and the Annuity Shopper. It is the best source of information I have found on the subject of annuities. And a special thanks for calling and answering my questions.
>
> As an attorney who frequently represents clients regarding investments, retirement and estate planning I will highly recommend you for their annuity needs. As I mentioned I have several annuities now and will continue to add more to my own retirement portfolio.
>
> I have recommended you and your company to other estate planners and financial advisors.
>
> Very truly yours, David D. Dunakey

Similar testimonies are frequently published.

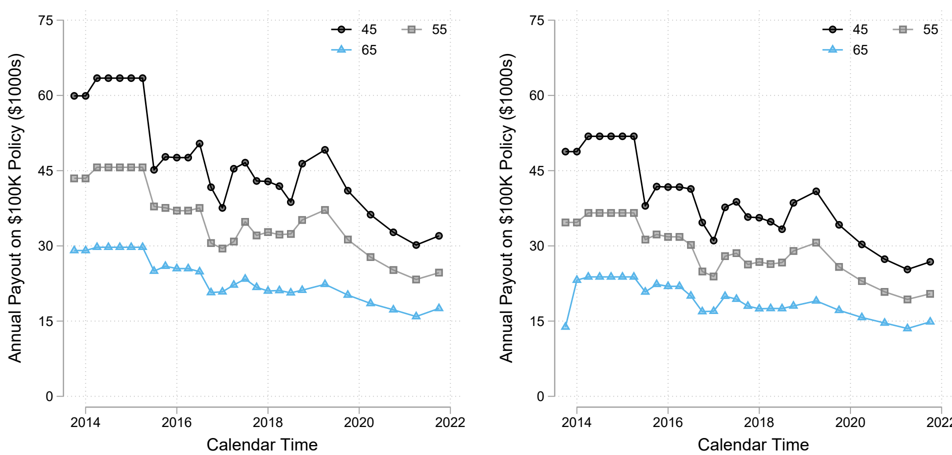
Sales of Annuity Products. Figure A1 shows how the size of this market has evolved over time with the left panel displaying new deferred income annuity sales. At the start of this period the market for deferred income annuities was small, totaling around $200 million and accounting for less than 1% of the overall fixed income annuity market. Subsequently, however, the market expanded rapidly between 2012 and 2014, with the real value of new considerations more than quadrupling in size. The right panel shows sales of all other fixed income annuity products, the most similar alternative product in



(A) Deferred Income Annuities (B) Other Fixed Income Annuities

FIGURE A1. Growth In The Deferred Income Annuities Market

the annuity space. Comparing the two panels suggests an element of common cyclicality, but growth in fixed income annuities is much smaller over the period.



(A) New York Life (Men) (B) New York Life (Women)

FIGURE A2. Average Payouts from Deferred Income Annuity Policies Beginning Payment at Age 80 By Gender and Purchase Age

Within Insurer Payout Variation. Figure A2 repeats the exercise in Figure 5 for contracts offered by New York Life, but conditioning on the deferral age and varying the purchase age. The left panel shows contracts for men and the right panel for women with each line corresponding to a different purchase age between 45 and 65. Again, contracts that offer payouts further into the future have higher payout amounts. Similarly, payouts for men who have lower expected longevity at all ages are smaller than for women, Despite systematic differences by product characteristics, a notable feature of both Figures 5 and A2 is evidence of a market level trend in payouts over time for all contract terms and for both insurers. This reflects that the largest cost to the insurer is the opportunity cost of foregone investment (see Mitchell et al. 1999) as well as a changing pool of annuitants.

B.1. Additional Empirical Results for Section 3

	(1) Base	(2) Time FEs	(3) Levels	(4) IAM Life Table	(5) Policy FEs
$\mathbb{1}[a > 45] \ln Q_t$	-0.238^{***}	-0.245^{***}	-0.046^{***}	-0.064^{***}	-0.234^{***}
	(0.027)	(0.024)	(0.005)	(0.008)	(0.027)
N	5,857	5,857	5,882	5,882	5,857
R^2	0.514	0.653	0.666	0.395	0.522

Notes: * $p < 0.10$, ** $p < 0.5$, *** $p < 0.01$. Huber-White heteroskedasticity robust standard errors are reported. Estimates reported for coefficient δ in equation (20). Implied selection is recovered using equation (19) and panel data collected from policy quotes printed in *Annuity Shopper* magazine. Aggregate quantities correspond to the cumulative real value of new deferred income annuity considerations collected from LIMRA and new considerations for all other fixed income annuity (FIA) products reported in Figure 4. All Specifications additionally control for purchase age indicators, insurer specific indicators, gender, deferral age indicators and aggregate demand for all other FIA products (excluding (2) due to colinearity with the time specific fixed effects).

TABLE A3. Adverse Selection and Market Maturity

Table A3 presents alternative specifications corresponding to the results in Table 1. The first column repeats the baseline estimate from the final specification with the full set of controls in Table 1. The second column allows for a non-parametric time trend by replacing the control for contemporaneous demand in all other fixed income annuity markets with a series of quarterly indicators. The parameter estimate is almost unchanged, highlighting that identification does not rely on the parametric control for aggregate market conditions.

The third column replaces the dependent variable with selection measured as deviations in levels rather than logs. The magnitude of the estimate changes due to the alternative specification of the dependent variable, but the qualitative patterns remain. The fourth column uses life tables from the individual Annuitant Mortality (IAM) 2012 table, which is described in American Academy of Actuaries / Society of Actuaries (2011). As discussed by Poterba and Solomon (2021) this forecasts significant mortality improvement at odds with observed mortality improvements. Consequently, the overall money's worth measure of annuities is more positive and the log specification discards a large share of the observations where the implied NPV exceeds the cost to consumers. Nevertheless, results across columns 3 and 4, both using levels, suggest that there are similar qualitative findings whether deviations are measured from the pool of all potential consumers or the pool of typical annuitants.

Finally, the fifth column replaces the controls for policy characteristics with a finer set of interact-characteristic fixed effects. Each annuity policy is identified by an insurer-gender-deferral age specific identifier and this specification controls for fixed effects along this interacted dimension. This allows for more granular differences in how insurers market to or administer plans by gender and deferral age which may drive estimates of the effect of maturity due to differential entry and exit of insurers and policy combinations. The point estimate is almost identical to the baseline specification.

Table A4 repeats this exercise for the results reported in Table 2 which allow for heterogeneous effects by purchase age. Once again the results are robust to alternative controls and definitions of the dependent variable. Point estimates for specifications with alternative controls are almost identical and alternative specifications of the dependent variable yield similar qualitative findings that echo results reported in Table 1. Across all specifications in both Tables A3 and A4 the estimated effects remain economically significant and statistically significant at the 1% level.

	(1) Base	(2) Time FEs	(3) Levels	(4) IAM Life Table	(5) Policy FEs
$\mathbb{1}[a = 55] \ln Q_t)$	-0.171^{***}	-0.176^{***}	-0.031^{***}	-0.041^{***}	-0.168^{***}
	(0.029)	(0.024)	(0.005)	(0.009)	(0.028)
$\mathbb{1}[a = 65] \ln Q_t$	-0.323^{***}	-0.334^{***}	-0.065^{***}	-0.093^{***}	-0.320^{***}
	(0.027)	(0.025)	(0.005)	(0.010)	(0.027)
N	5, 857	5, 857	5, 882	5, 882	5, 857
R^2	0.519	0.658	0.669	0.399	0.527

Notes: * $p < 0.10$, ** $p < 0.5$, *** $p < 0.01$. Huber-White heteroskedasticity robust standard errors are reported. Estimates reported for coefficient $\delta_5 5$ and $\delta_6 5$ in equation (21). Implied selection is recovered using equation (19) and panel data collected from policy quotes printed in *Annuity Shopper* magazine. Aggregate quantities correspond to the cumulative real value of new deferred income annuity considerations collected from LIMRA and new considerations for all other fixed income annuity (FIA) products reported in Figure 4. All Specifications additionally control for purchase age indicators, insurer specific indicators, gender, deferral age indicators and aggregate demand for all other FIA products (excluding (2) due to colinearity with the time specific fixed effects).

TABLE A4. Adverse Selection and Market Maturity by private information

Appendix C. Proofs & Additional Results for Section 4.1

PROOF OF PROPOPSITION 4 . Proof of Part (i). Beginning with the monotonicity of marginal costs – the CDF of the conditional random variable $\pi|\omega = p$ is

$$F_{\pi|\omega}(\pi|\omega) \equiv \int_{-\infty}^{\pi} f_{\pi|\omega}(u|\omega)du = \int_{-\infty}^{\pi} \frac{f_{\pi,\omega}(u,\omega)}{f_{\omega}(\omega)}du$$
$$= \widehat{C}_2(u, F_{\omega}(\omega)) = g(F_{\pi}(\pi)), \tag{A1}$$

for a given copula $\widehat{C}$ where $\widehat{C}_2(x, y) = \partial\widehat{C}(x,y)/\partial y$.

As the marginal cost curve is a conditional expectation $MC(p) = E(\pi|\omega = p)$, stochastic dominance of the conditional distributions is a sufficient condition to order marginal costs in price

$$F_{\pi|\omega}(\pi|\omega = v_1) \succeq F_{\pi|\omega}(\pi|\omega = v_2) \;\; \forall v_1 \geq v_2 \tag{A2}$$

$$\longrightarrow MC(v_1) = E(\pi|\omega = v_1) \geq E(\pi|\omega = v_2) = MC(v_2) \;\; \forall v_1 \geq v_2 \;. \tag{A3}$$

Substituting equation (A1) gives the following necessary and sufficient condition:

$$\widehat{C}_2(u, F_{\omega}(v_1)) \leq \widehat{C}_2(u, F_{\omega}(v_2)) \; \forall v_1 \geq v_2 \textit{ and } \forall u \tag{A4}$$

$$\longleftrightarrow \widehat{C}_2(u, a) \leq \widehat{C}_2(u, b) \; \forall a \geq b \textit{ and } \; \forall u \tag{A5}$$

as $v_1 \geq v_2 \longrightarrow F_{\omega}(v_1) \geq F_{\omega}(v_2)$. This is satisfied for all copulas where $C_2(\cdot,\cdot)$ is decreasing in its second argument. Condition (i) of Proposition (4) guarantees $\widehat{C}$ is concave and satisfies this property.

This establishes that $MC(p)$ is increasing in p.

Proof of Part (ii). Turning to the monotonicity of average costs – the CDF of the conditional random variable $\pi|\omega > p$ is

$$F_{\pi|\omega > p}(\pi|\omega > p) \equiv \frac{u - \widehat{C}(u, F_\omega(p))}{1 - F_\omega(p)}, \tag{A6}$$

for a given copula $\widehat{C}$.

As the average cost curve is defined as a conditional expectation $AC(p) = E(\pi|\omega \geq p)$, stochastic dominance of the conditional distributions is a sufficient condition to order average costs in price

$$F_{\pi|\omega \geq p}(\pi|\omega \geq v_1) \succeq F_{\pi|\omega \geq p}(\pi|\omega \geq v_2) \; \forall v_1 \geq v_2 \tag{A7}$$

$$\longrightarrow AC(v_1) = E(\pi|\omega \geq v_1) \geq E(\pi|\omega \geq v_2) = AC(v_2) \; \forall v_1 \geq v_2 \tag{A8}$$

Substituting equation (A6) gives the following necessary and sufficient condition:

$$F_{\pi|\omega \geq p}(\pi|\omega \geq v_1) \leq F_{\pi|\omega \geq p}(\pi|\omega \geq v_2) \; \forall v_1 \geq v_2 \; \textit{and} \; \forall \pi \tag{A9}$$

$$\frac{u - \widehat{C}(u, F_\omega(v_1))}{1 - F_\omega(v_1)} \leq \frac{u - \widehat{C}(u, F_\omega(v_2))}{1 - F_\omega(v_2)} \; \forall v_1 \geq v_2 \; \textit{and} \; \forall u \tag{A10}$$

as $v_1 \geq v_2 \longrightarrow F_\omega(v_1) \geq F_\omega(v_2)$, condition (ii) of Proposition (4) guarantees $\widehat{C}$ satisfies this property, delivering an ordering of average costs for a given copula.

□

PROOF OF LEMMA 1 . It is widely known that monotone likelihood ratio dependence is a stronger dependence property than those implied by the conditions in Proposition 4 (see, e.g., Nelsen 2006 Theorem 5.2.19). □

C.1. Additional Results and Discussion of Copula

This section shows how to derive the distribution of willingness-to-pay directly from assumptions on cost and risk premia. This will depend on the dependence between cost and risk premia, as encoded by the copula C, and marginal distributions. The following results are derived in Cherubini et al. (2011) who also provide extensions to multivariate settings (see also Navarro and Sarabia 2022). These multivariate extensions can also be used to derive expressions for the joint-density of net premia from the underlying densities of risk premia r and choice frictions ε. This extends general results on the convolution of probability distributions.

The distribution function for the ω is given by F_ω

$$F_\omega(\omega) = \int_{-\infty}^{\infty} f_\pi(c)\partial_1 C(F_\pi(c), F_{\tilde{r}}(\omega - c))\, dc \tag{A11}$$

provided that $\lim_{v\to 0^+} \partial_1 C(u, v) = 0$ for all $u \in (0, 1)$. This is a mixture over the conditional distribution of $(\tilde{r} + c|\pi = c)$, which is given by[1]

$$F_{\omega|\pi}(\omega|\pi = c) = \partial_1 C(F_\pi(c), F_{\tilde{r}}(\omega - c)). \tag{A12}$$

We can use this to derive the following joint density function

$$F_{\pi,\omega}(t, v) = \int_{-\infty}^{t} F_{\omega|\pi}(v|t)\, dF_\pi(t) = \int_{-\infty}^{t} \partial_1 C(F_\pi(t), F_{\tilde{r}}(v - t))\, dF_\pi(t), \tag{A13}$$

which in turn can be used recover $\hat{C}$, the copula specifying the dependence between ω and π, by application of Sklar's Theorem:

$$\hat{C}(u, v) = \int_{-\infty}^{v} \partial_1 C(x, F_{\tilde{r}}(F_\omega^{-1}(u) - F_\pi^{-1}(x)))\, dx. \tag{A14}$$

[1]Theorem 2.3 Navarro and Sarabia 2022 shows that this admits a distortion function representation.

In this general form, there are not closed form results for this integral as it depends on the marginal densities and the copula. However, we can additionally derive the following probability density functions which will prove useful in evaluating its properties:

$$f_{\omega|\pi}(\omega|\pi = c) = f_{\tilde{r}}(\omega - c)\partial_{1,2}C(F_\pi(c), F_{\tilde{r}}(\omega - c)) \tag{A15}$$

$$f_\omega(\omega) = \int_{-\infty}^{\infty} f_\pi(c) f_{\tilde{r}}(\omega - c)\partial_{1,2}C(F_\pi(c), F_{\tilde{r}}(\omega - c))\, dc \tag{A16}$$

and the following corollary uses these representations to re-express Lemma 1 in terms of the joint distributions of π and $\tilde{r}$.

COROLLARY A1 (MLRP). *The monotone-likelihood ratio property for willingness-to-pay, ω, and cost type, π, is equivalent to the following property for the cost types and risk premia $\tilde{r}$:*
$\frac{f_{\tilde{r}}(r-\pi)\partial_{1,2}C(F_\pi(\pi),F_{\tilde{r}}(r-\pi))}{f_{\tilde{r}}(r-\pi')\partial_{1,2}C(F_\pi(\pi'),F_{\tilde{r}}(r-\pi'))} \geq \frac{f_{\tilde{r}}(r'-\pi)\partial_{1,2}C(F_\pi(\pi),F_{\tilde{r}}(r'-\pi))}{f_{\tilde{r}}(r'-\pi')\partial_{1,2}C(F_\pi(\pi'),F_{\tilde{r}}(r'-\pi'))}\ \forall \pi' \geq \pi$ *and* $r' \geq r$

Thus, all joint distributions satisfying this property satisfy conditions (i) and (ii) in Proposition 6.

I now turn to discussing examples of common parametric copulas and the conditional distributions of ranks (summarized by their derivatives). While this allows for convenient theoretical analysis, it is worth emphasizing, however, that in empirical settings copulas can be estimated directly from the data using non-parametric tools. This is an inexhaustive list and other copula family also abound.[2,3]

Example 1: Gaussian copula. The Gaussian copula is a family of copulas that vary in their correlation parameter ρ. All multivariate Normal distributions have Gaussian

[2]Moreover, the convolution of copulas above is closed under mixtures which, with sufficient mixing, allows for almost arbitrary dependence using parametric families.

[3]A less common copula in statistical applications, but highly relevant in economic applications is the Burr copula. This is characterized by joint-Burr distributions which exhibit similar jointness properties to the normal distribution. Moreover, univariate Burr distributions have extensive use as a generalized version of the Pareto distribution to model income distributions with long tails (see, for instance, Singh and Maddala 1976).

copulas and Normal marginals. However, not all normally distributed marginals have Gaussian copulas. The bivariate copula is defined as:

$$C(u,v;\rho) = \Phi^{-1}(u)\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{\Phi^{-1}(v)} \exp\left[-\frac{1}{2}\frac{y^2 - 2\rho\Phi^{-1}(u)\,y + \Phi^{-1}(u)^2}{1-\rho^2}\right] dy \tag{A17}$$

$$= \Phi^{-1}(u)\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\Phi^{-1}(u)^2}{2}\right]\Phi\left[\frac{\Phi^{-1}(v) - \rho\Phi^{-1}(u)}{\sqrt{1-\rho^2}}\right] \tag{A18}$$

with first derivative given by (it is symmetric)

$$\frac{d}{du}C(u,v;\rho) = \frac{d}{du}\Phi^{-1}(u)\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\Phi^{-1}(u)^2}{2}\right]\Phi\left[\frac{\Phi^{-1}(v) - \rho\Phi^{-1}(u)}{\sqrt{1-\rho^2}}\right] \tag{A19}$$

$$= \Phi\left[\frac{\Phi^{-1}(v) - \rho\Phi^{-1}(u)}{\sqrt{1-\rho^2}}\right], \tag{A20}$$

and cross-partial derivative (or copula density)

$$\frac{\partial^2}{\partial u \partial v}C(u,v;\rho) = \frac{1}{\sqrt{1-\rho^2}}\frac{d}{dv}\Phi^{-1}(v)\Phi'\left[\frac{\Phi^{-1}(v) - \rho\Phi^{-1}(u)}{\sqrt{1-\rho^2}}\right] \tag{A21}$$

$$= \frac{1}{\sqrt{1-\rho^2}}\exp\left[\frac{-\rho^2\Phi^{-1}(v)^2 + 2\rho\Phi^{-1}(u)\Phi^{-1}(v) - \rho^2\Phi^{-1}(u)^2}{2\left(1-\rho^2\right)}\right]. \tag{A22}$$

Substituting into Equation A12 gives the following expression for the joint density in this example:

$$F_{\omega|\pi}(\omega|\pi = c) = \Phi\left[\frac{\Phi^{-1}(F_\pi(c)) - \rho\Phi^{-1}(F_{\tilde{r}}(\omega - c))}{\sqrt{1-\rho^2}}\right], \tag{A23}$$

which highlights that the normality of the conditional density is a special property. It requires both the specific Gaussian copula and the normally distributed marginals.

Figure A3 shows how the parametric specification of the copula effects the joint distribution when the marginal distribution are fixed. The left columns shows how the joint distribution of π and $\tilde{r}$ changes with the correlation parameter. In both cases I assume π and $\tilde{r}$ are log normally distributed with their logs following standard normals. Comparing panels A and C shows the role of the correlation parameter for the gaussian copula. As the correlation increases, it stretches the density towards the upper right corner.

Example 2: Gumbel Copula. The Gumbel copula has heavier tails than the normal distribution and is also asymmetric, with more weight in the right tail. It's left tail, however, is similar to the normal distribution. The bivariate copula is defined (for $\delta \geq 1$) as:

$$C(u, v; \delta) = \exp\left(-\left[(-\log u)^{\delta} + (-\log v)^{\delta}\right]^{1/\delta}\right) \tag{A24}$$

$$= \exp\left(-A^{1/\delta}\right) \quad \text{where} \quad A = (-\log u)^{\delta} + (-\log v)^{\delta} \tag{A25}$$

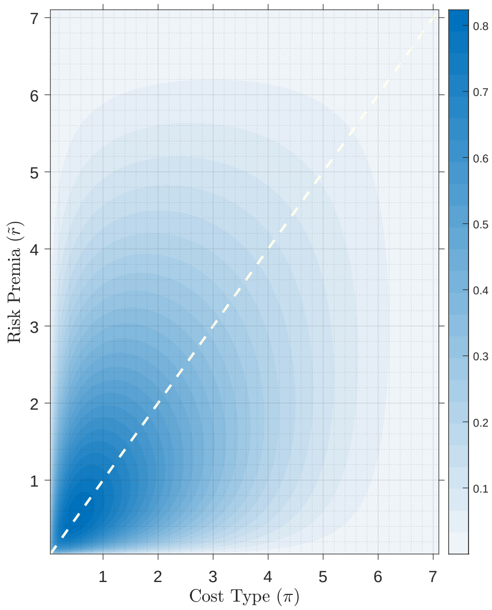
with first derivative given by (it is symmetric)

$$\frac{\partial}{\partial u} C(u, v; \delta) = \exp\left(-A^{1/\delta}\right) \cdot \frac{(1 + (-\log v/-\log u)^{\delta})^{1/\delta - 1}}{u} \tag{A26}$$

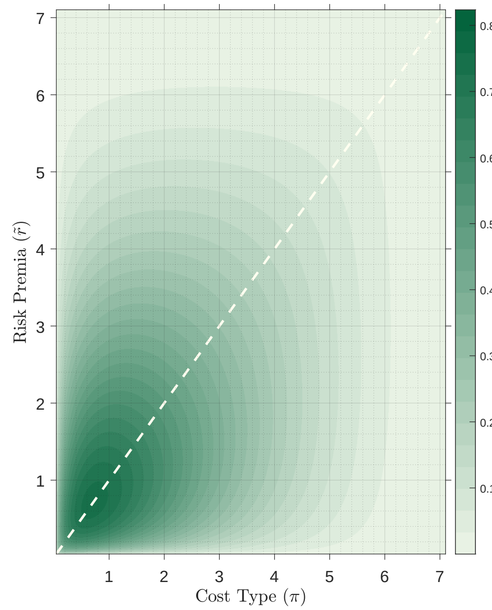
and cross-partial derivative

$$\frac{\partial^2}{\partial u \partial v} C(u, v; \delta) = ot \frac{(-\log v)^{\delta-1}}{v} \left[\frac{1-\delta}{\delta}(A^{1/\delta} + \delta - 1)A^{1/\delta - 2}\right]. \tag{A27}$$

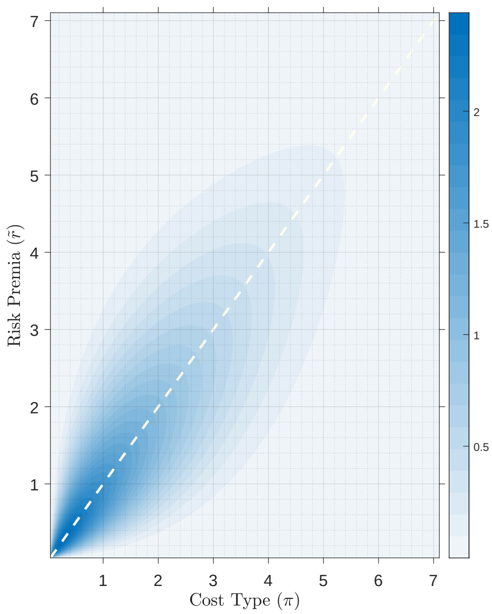
Comparing the right column of Figure A3, which shows the Gumbel copula, to the left highlights differences induced in the joint density by choice of the copula. At lower correlations, panels A and B, the two copulas deliver more similar joint-densities. However, panel B, using the Gumbel copula, reveals heavier tails of the joint distribution. This is even more pronounced for higher correlations shown in panels C and D. Here, like the Gaussian copula , panel D stretches the density towards the upper-right corner



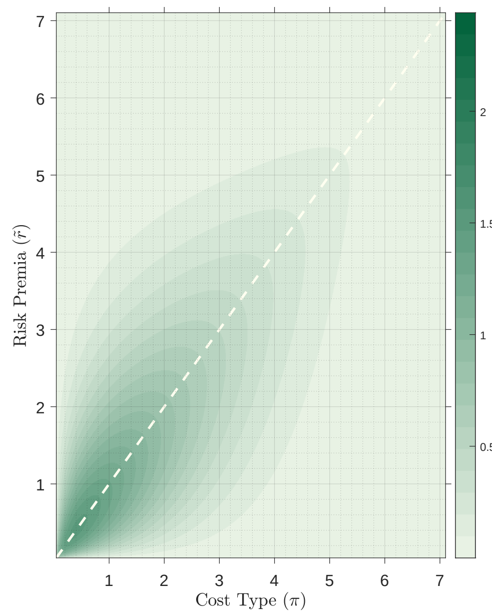
(A) Gaussian Copula (correlation=0.5)



(B) Gumbel Copula (correlation=0.5)



(C) Gaussian Copula (correlation=0.9)



(D) Gumbel Copula (correlation=0.9)

FIGURE A3. Copula and Joint Densities

Notes: Panels vary the copula (and parameter controlling correlation) for the rank dependency between cost type (π) and risk premia ($\tilde{r}$). In all panels the marginal distributions of cost type (π) and risk premia ($\tilde{r}$) are distributed log-normally with a mean of 0 and unit variance.

of the figure. In contrast though, the normal exhibits softer contours. Instead, the surface of the joint density is pulled "tauter" under the Gumbel copula and more mass is concentrated in the joint tail. Thus the risk is more jointly heavily tailed.

Example 3: The Independent Copula. The independent copula describes the case where the two random variables are independent. The relevant formulae are:

$$C(u, v) = uv \tag{A28}$$

$$\frac{\partial}{\partial u} C(u, v) = v \tag{A29}$$

$$\frac{\partial^2}{\partial u \partial v} C(u, v) = 1. \tag{A30}$$

Appendix D. General Proofs for Section 4.2

PROOF OF PROPOPSITION 5. First consider linear utility with $\omega_i = \pi_i^{IM} + \tilde{r}_i^{IM}$. Assumption 3 and Assumption 1 guarantee that the conditions in Theorem 3.3 in Navarro and Sarabia (2022) are satisfied. A direct application gives

$$\omega_i^{IM} = \pi_i^{IM} + r_i^{IM} + \varepsilon_i^{IM} \preceq \pi_i^{MM} + r_i^{MM} + \varepsilon_i^{MM} = \omega_i^{MM} \longrightarrow F_\omega^{MM}(p) \leq F_\omega^{IM}(p) \,\forall\, p\,. \quad \text{(A31)}$$

As the usual stochastic order is preserved under increasing transformations, the stated proposition then follows immediately for any increasing $u(\cdot)$ □

PROOF OF LEMMA 2. Identical in distribution implies $\pi^{MM} \succeq \pi^{IM}$. Thus, each element of the vector $(\pi_i, \tilde{r}_i)$ in mature markets is larger in the usual stochastic order than in immature markets. It follows that the vector $\mathbf{X}^{IM} \preceq \mathbf{X}^{MM}$ in the usual *multivariate* stochastic ordering (Shaked and Shanthikumar 2007, Theorem 6.B.14).

This order is preserved for any increasing function $u : \mathcal{R}^2 \to \mathcal{R}$, then $u(\mathbf{X}^{IM}) \preceq u(\mathbf{X}^{MM})$. Thus, $F_\omega^{IM}(p) \geq F_\omega^{MM}(p) \,\forall\, p$ and the stated lemma follows. This result extends to any multi-dimensional vector (for example r and ε) under corresponding assumptions on their marginals. □

PROOF OF LEMMA 3. Analogously to the proof of Proposition 4

$$F_{\pi|\omega}^{IM}(\pi|\omega) \succeq F_{\pi|\omega}^{MM}(\pi|\omega) \longrightarrow MC^{IM}(p) \geq MC^{MM}(p) \,\forall\, p. \quad \text{(A32)}$$

As the unconditional first order stochastic dominance of private valuations in mature and immature markets implies $F_\omega^{MM}(\omega) \leq F_\omega^{IM}(\omega)$, (A32) holds if and only if the copulas satisfy

$$\widehat{C}_2^{MM}(u, a) \leq \widehat{C}_2^{IM}(u, b) \;\forall a \geq b \textit{ and } \forall u. \quad \text{(A33)}$$

This is trivially satisfied under identical concave copulas ($\widehat{C} = \widehat{C}^{IM} = \widehat{C}^{MM}$). However, it cannot be satisfied for different copulas that are both concave in their second argument (Proposition 4 condition(i)). □

PROOF OF PROPOSITION 7. Recall the following necessary and sufficient condition for the comparison of average costs (See the proof of Proposition 4):

$$F_{\pi|\omega\geq p}(\pi|\omega \geq v_1) \leq F_{\pi|\omega\geq p}(\pi|\omega \geq v_2)\ \forall v_1 \geq v_2 \text{ and } \forall\pi \tag{A34}$$

$$\frac{u - \widehat{C}(u, F_\omega(v_1))}{1 - F_\omega(v_1)} \leq \frac{u - \widehat{C}(u, F_\omega(v_2))}{1 - F_\omega(v_2)}\ \forall v_1 \geq v_2 \text{ and } \forall u \tag{A35}$$

The analogous condition to order average costs between markets is:

$$F^{IM}_{\pi|\omega\geq p}(\pi|\omega \geq v) \leq F^{MM}_{\pi|\omega\geq p}(\pi|\omega \geq v)\ \forall v, \pi \tag{A36}$$

$$\frac{u - \widehat{C}^{IM}(u, F^{IM}_\omega(v))}{1 - F^{IM}_\omega(v)} \leq \frac{u - \widehat{C}^{MM}(u, F^{MM}_\omega(v))}{1 - F^{MM}_\omega(v)}\ \forall v, \forall u \tag{A37}$$

As $F^{MM}_\omega(\omega) \leq F^{IM}_\omega(\omega)$, established in Proposition 5, the following is true for all copulas satisfying condition (ii) of Proposition 4 (see the proof above)

$$\frac{u - \widehat{C}^{MM}(u, F^{IM}_\omega(v))}{1 - F^{IM}_\omega(v)} \leq \frac{u - \widehat{C}^{MM}(u, F^{MM}_\omega(v))}{1 - F^{MM}_\omega(v)}\ \forall v \text{ and } \forall u, \tag{A38}$$

and by the concordance ordering in Assumption 3 we have

$$\frac{u - \widehat{C}^{IM}(u, F^{IM}_\omega(v))}{1 - F^{IM}_\omega(v)} \leq \frac{u - \widehat{C}^{MM}(u, F^{IM}_\omega(v))}{1 - F^{IM}_\omega(v)}\ \forall v \text{ and } u \in [0,1], \tag{A39}$$

which completes the proof. □

PROOF OF PROPOSITION 8. Note that the copula defines *rank* dependence. Consequently, they are invariant under strictly monotone transformations, including the inverse demand transformation considered here. Thus, to order average costs given quantity, q, we have the following condition:

$$\frac{u - \widehat{C}^{IM}(u,q)}{1-q} \leq \frac{u - \widehat{C}^{MM}(u,q)}{1-q} \; \forall q \in [0,1) \textit{ and } u \in [0,1], \tag{A40}$$

which simplifies to

$$\widehat{C}^{IM}(u,q) \geq \widehat{C}^{MM}(u,q) \; \forall q \in [0,1) \textit{ and } u \in [0,1]. \tag{A41}$$

This only depends on the change in sorting ($\widehat{C}^{IM} \neq \widehat{C}^{MM}$) and holds with equality under no re-sorting ($\widehat{C} = \widehat{C}^{IM} = \widehat{C}^{MM}$). The stated Proposition then follows from the concordance ordering in Assumption 3.

Equation (A33) gives an identical condition for marginal costs in terms of the conditional density which corresponds to stochastic dominance

$$\widehat{C}_2^{IM}(u,q) \geq \widehat{C}_2^{MM}(u,q) \; \forall q \in [0,1) \textit{ and } u \in [0,1]. \tag{A42}$$

This cannot be satisfied as stochastic dominance for all conditional distributions implies stochastic dominance of the unconditional distribution. This violates assumption 2.

Note that at $q = 0$ marginal costs are equal to average costs. Then, the monotonicity of marginal costs and the results of Proposition 6 imply a single crossing property which completes the proof.

□

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